Critical points in the nuclear matter phase diagram: the universality class of nuclear and quark-gluon matter.

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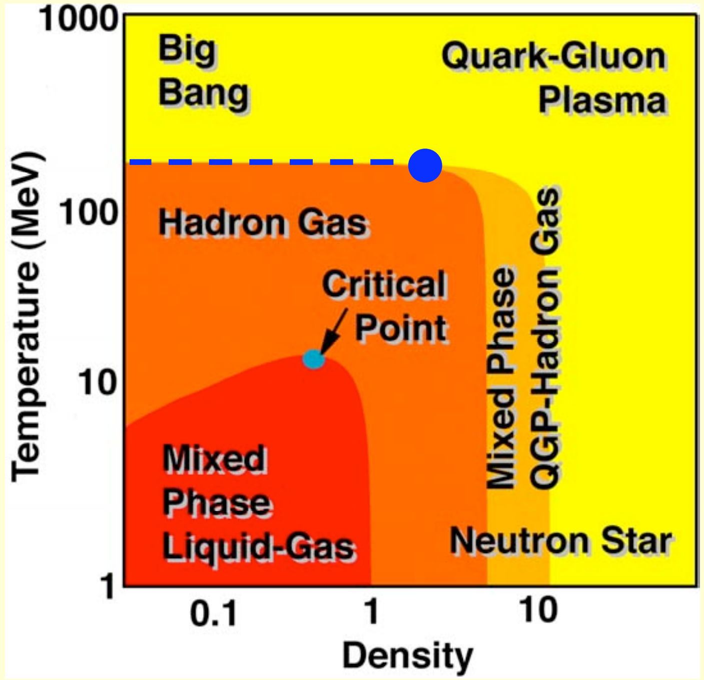
OUTLINE

- General remarks on phase diagrams
- State variables and how to measure them
- The role of isospin
- First order transitions
- Continuous transitions in finite multi-component systems
- Event-by-event analysis of fluctuations
- The ISiS data set, scaling analysis, and determination of critical exponents
- What will CBM@FAIR / RHIC low energy scan be able to measure?

Nuclear Matter Phase Diagram

- Two (at least) phase transitions in nuclear matter:
  - *"Liquid Gas"*
  - *Hadron gas⇔QGP / chiral restoration*
- Problems/ Opportunities:
  - *Finite size effects*
  - *Is there equilibrium?*
  - *Measurement of state variables (ρ, T, S, p, …)*
  - *Migration of nuclear system through phase diagram*



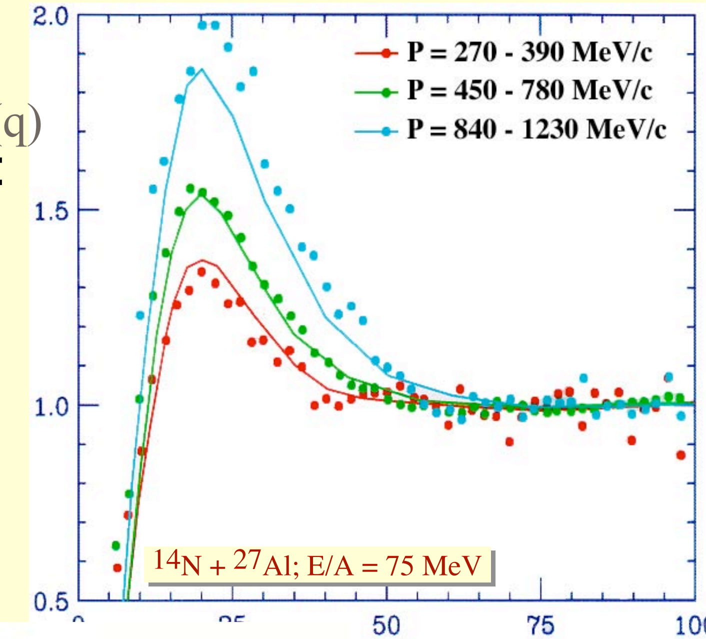
Source: NUCLEAR SCIENCE, A Teacher's Guide to the Nuclear Science Wall Chart, Figure 9-2

Volume

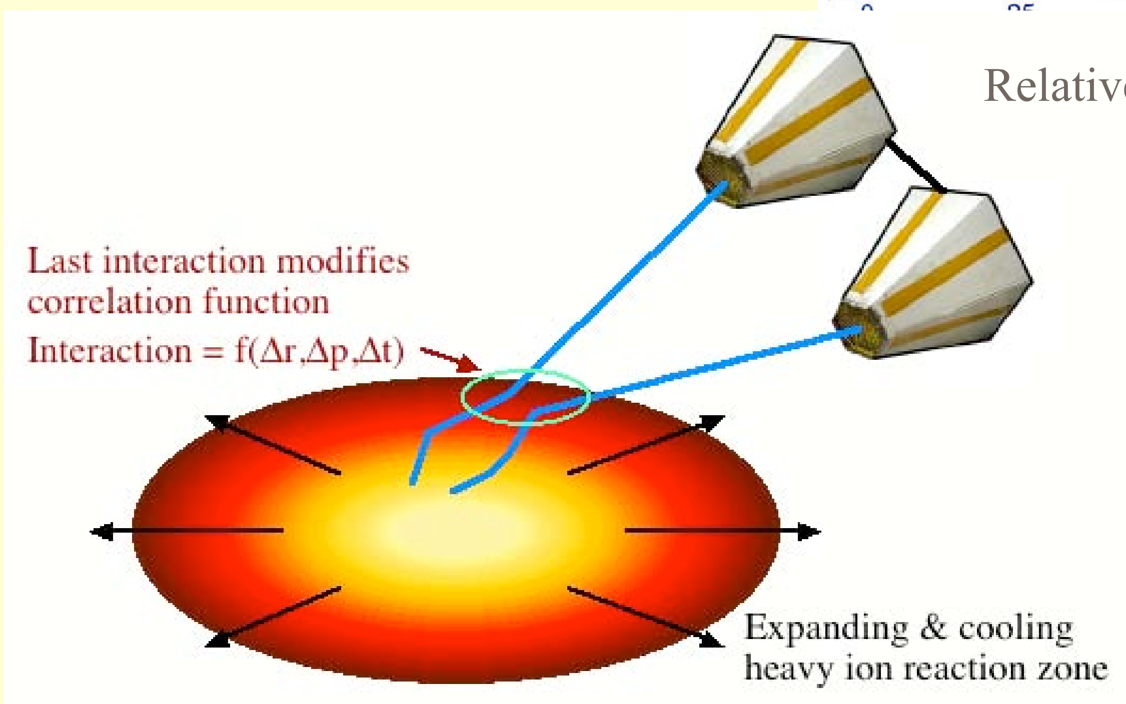
- Information from interferometry:
- Two-particle correlation are sensitive to space-time extension of emitting source

$$C(P,q)=\int d^3x\, F_p(r)\, |\phi(r,q)|^2$$

C(q)



Relative Momentum, q



Review: W.B., Gelbke, Pratt, Annu.Rev.Nuc.Part.Sci. 42

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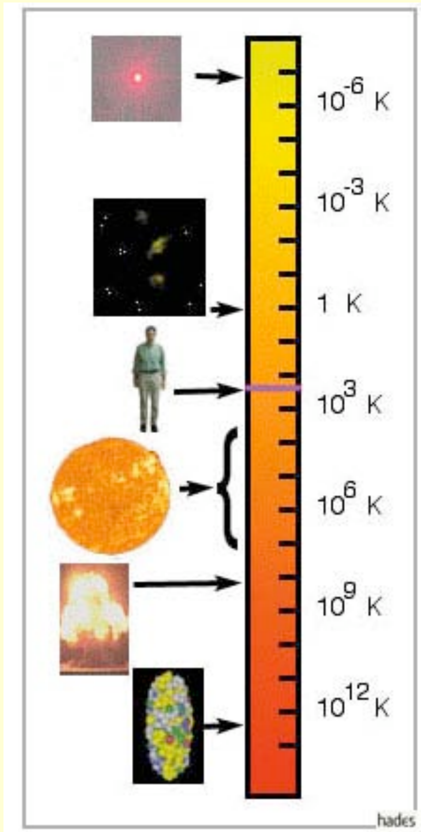
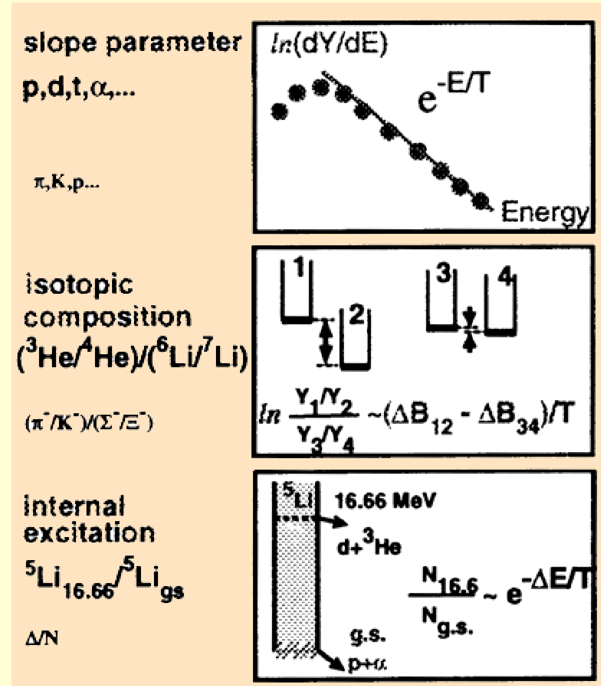
Temperature

J. Pochodzalla, CRIS '96

- Measure nuclear temperature indirectly via:



- *Slopes of charged particle spectra*
- *Bound-state populations*
- *Unbound states*
- *Fragment isotopic yields*



"He-Li" thermometer

Albergo et al., Nuovo C. A89

$$T = 13.3\ \mathrm{MeV}/\ln\left(2.18\frac{Y(^6Li)\,Y(^4He)}{Y(^7Li)\,Y(^3He)}\right)$$

Central question: At which ***time*** do we measure the temperature with each thermometer?

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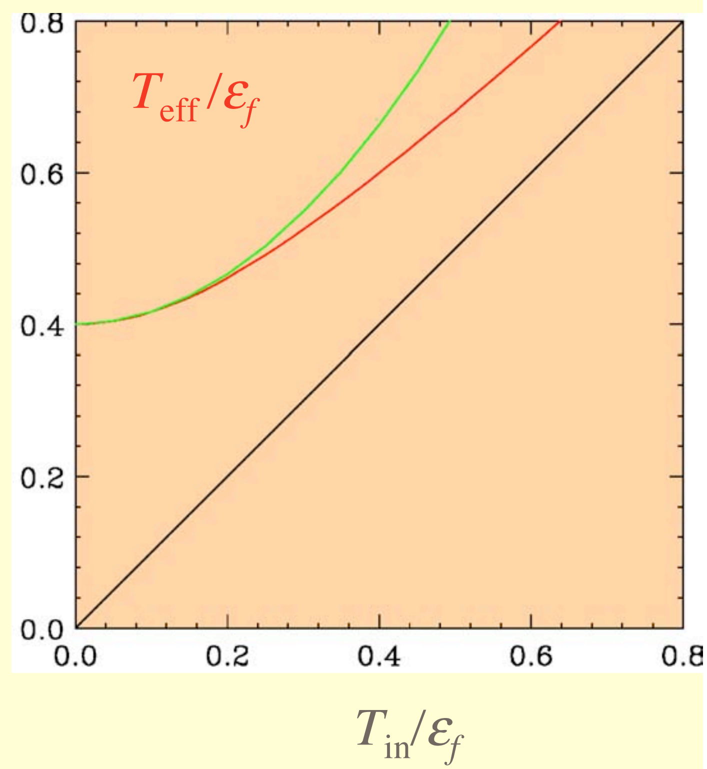
Temperature from Fragment Spectra

- Nucleon momentum distribution at temp. T: $$\rho(p) = \frac{1}{1+\exp[(p^2/2m-\mu)/T_{\rm in}]}$$
- Fragment momentum = sum of momenta of nucleons in it
- Problem equivalent to solving Pearson random walk in momentum space
- Limiting distribution: $$\rho(E_{\rm f}) = 2\sqrt{\frac{E_{\rm f}}{\pi\, T_{\rm eff}^3}}\exp\left(-\frac{E_{\rm f}}{T_{\rm eff}}\right)$$

  (Boltzmann with $T_{\rm eff} = \sigma^2/M_N$)
- Fragment slope "temperature", T_{eff}, is not equal to T, but is a monotonous function of it
  $\Rightarrow$ *Nuclear Thermometer*
- Approximation:

  $$T_{\rm eff} \approx (2/5)\epsilon_f\,(1+5\pi^2T^2/\epsilon_f^2+\ldots)$$

  WB, Phys. Rev. C **51**



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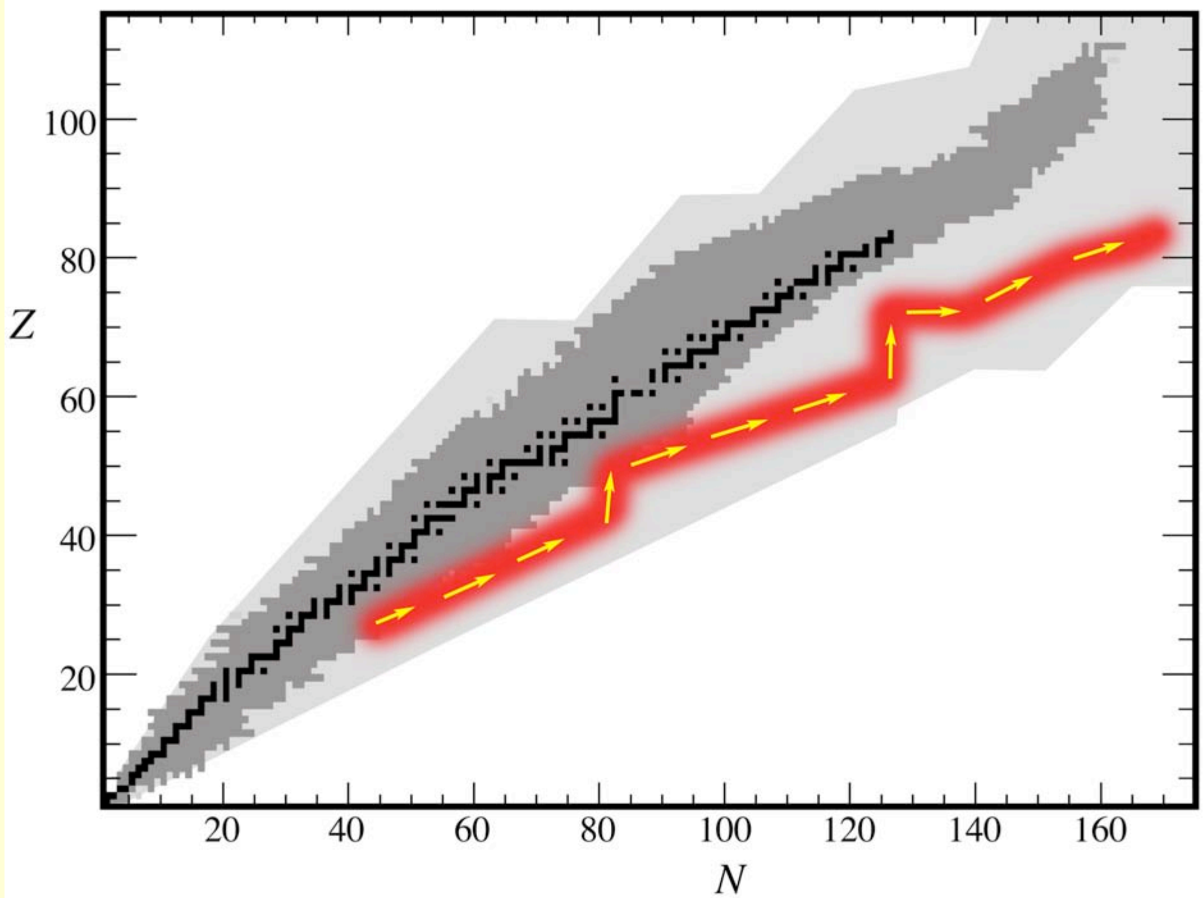
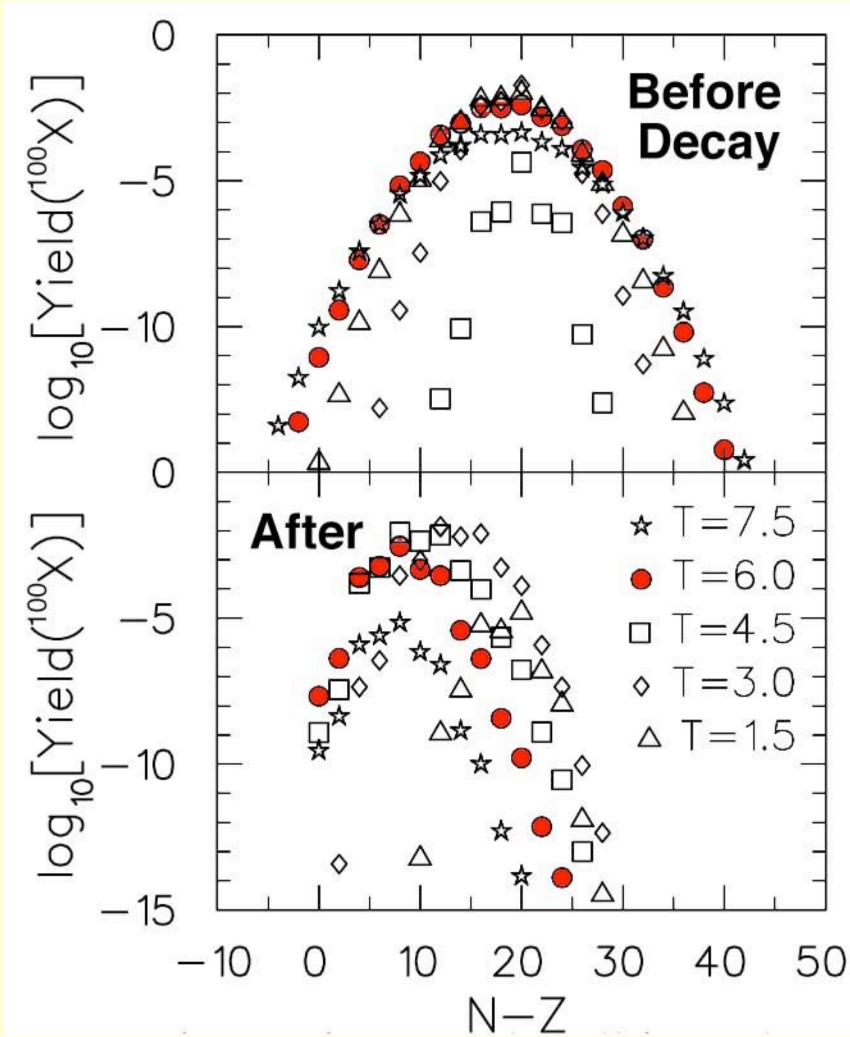
Isospin: RIB Reaction Physics

- Exploration of the drip lines below charge ~40 via projectile fragmentation reactions
- Determination of the isospin degree of freedom in the nuclear equation of state
- Astrophysical relevance
- Review: Li, Ko, WB, Int. J. Mod. Phys. E **7**



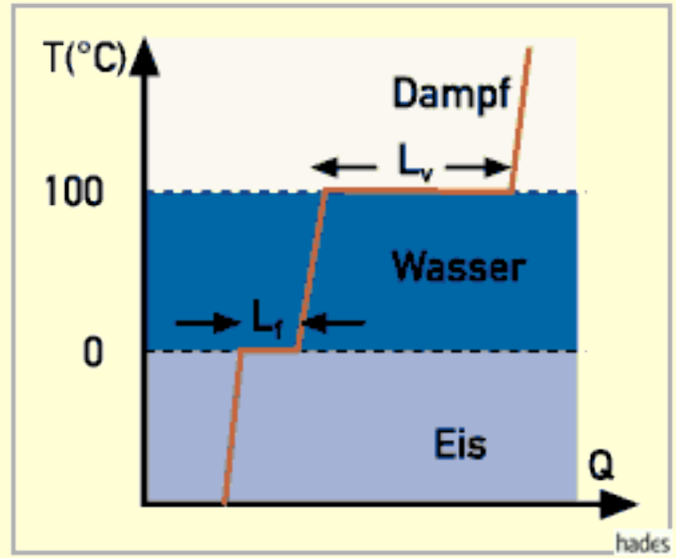
Width of Isotope Distribution, Sequential Decays



- Predictions for width of isotope distribution are sensitive to isospin term in nuclear EoS
- Complication: Sequential decay almost totally dominates experimentally observable fragment yields
  Pratt, WB, Morling, Underhill, PRC **63**

First-Order Phase Transition

- Coexistence of two phases (e.g. ice+water, water+steam)
- Addition of heat does not change temperature
  Latent heat
  (H_2O: L_f = 80 kcal/kg,
  L_v = 540 kcal/kg)
- Different *specific heat capacities* in the different phases -> different slopes T vs. Q
- *Pressure kept constant!*



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Benenson
Westfall
cliXX Physik
Verlag Harri Deutsch, Frankfurt 1999

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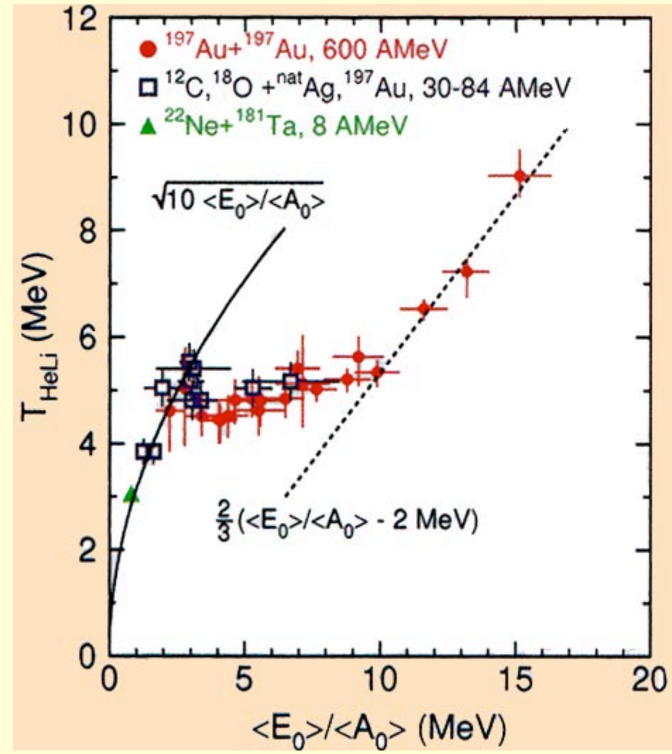
Observation of First-Order Phase Transition?

- Low E^*: Liquid-like
  $T \sim E^{*1/2}$
- High E^*: Gas-like
  $T \sim E^*$
- 1st order transition: Liquid-gas coexistence Temperature does not change in phase mixture while liquid is converted to vapor.
- Analogy: Boiling of water



J.Pochodzalla et al. (ALADIN), PRL **75**

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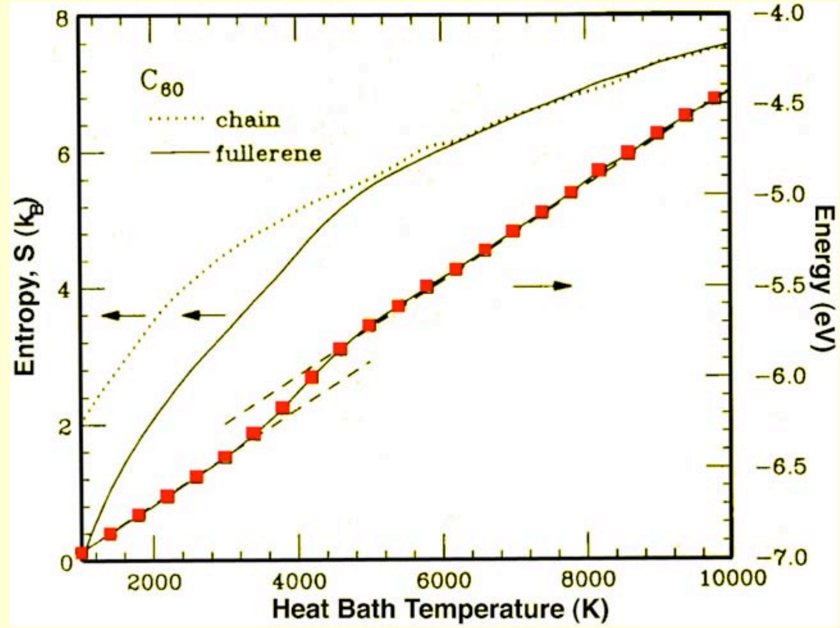
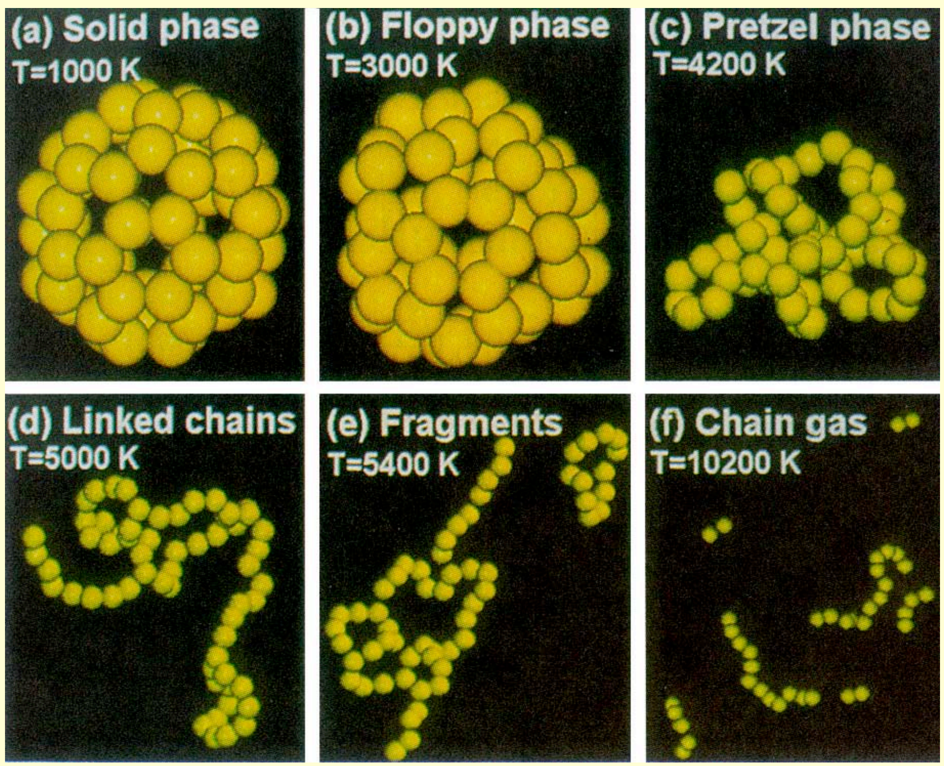
Buckyballs-Melting

- C_{60} Cluster
- Soccer Ball Geometry





- Molecular dynamics calculations
- Hoover-Nose heat bath

S.G. Kim & D. Tomanek, PRL 72

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Continuous Phase Transition

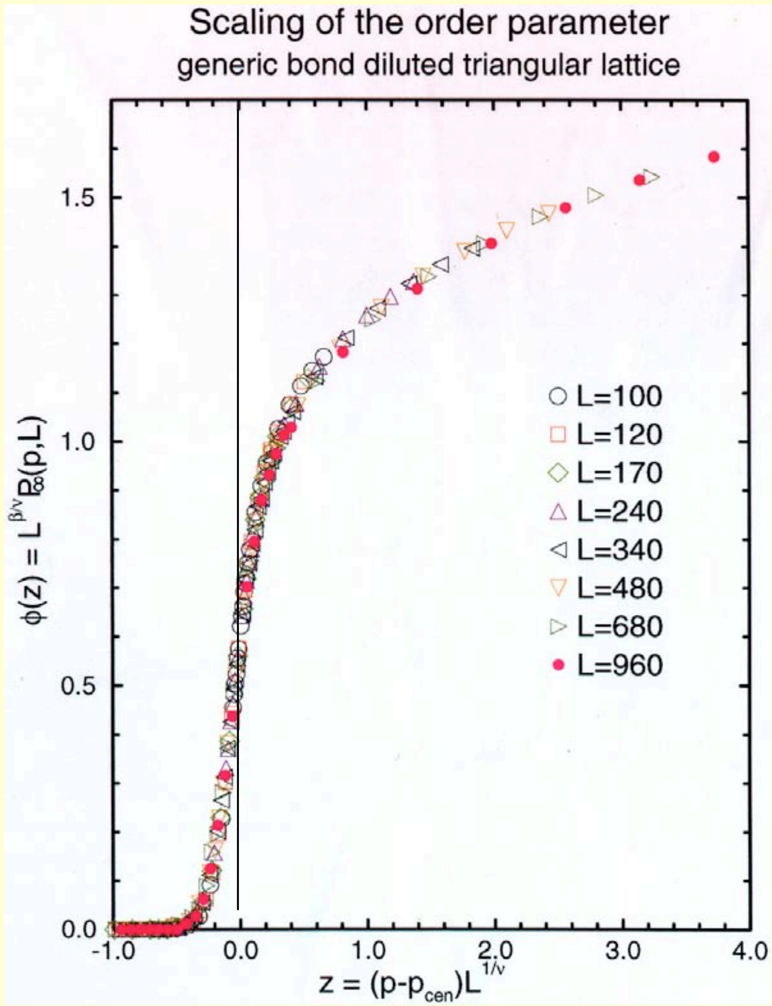
- Near critical point, we expect scaling behavior: all physical quantities have power-law dependencies on the control parameter
- No characteristic scales in observables
- Critical exponents of power-laws are main quantities of interest
  - τ, *Cluster size:* $n_s(T_c) \sim s^{-\tau}$
  - β, *Order parameter:* $P \sim (T-T_c)^{\beta}$
  - γ, *Divergence of* s: $\sim |T-T_c|^{-\gamma}$
- Hyper-scaling assumption

$$2 - \alpha = (\tau - 1) / \sigma = 2\beta + \gamma$$

Finite Size Scaling

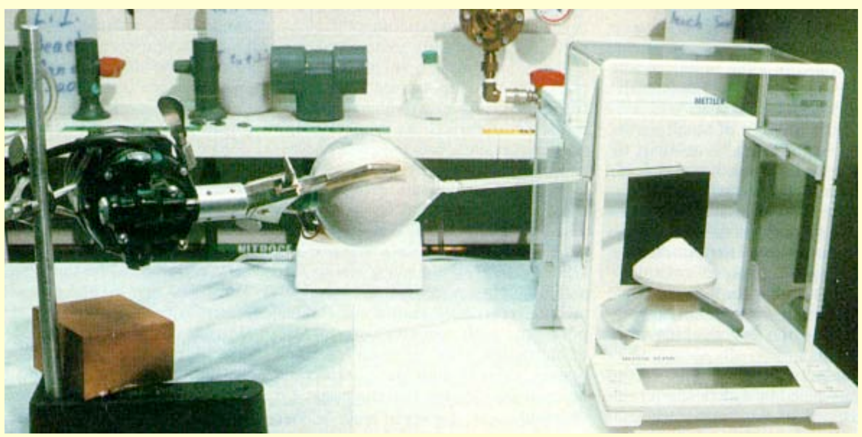
- Phase transitions strictly only defined for (almost) infinite systems
- Lattice calculations work on finite lattices and extrapolate to infinite lattices
- Finite size scaling exponent, ν
  - *Modify control parameter by* $L^{1/\nu}$
  - *Modify order parameter by* $L^{\beta/\nu}$
- Opportunity for nuclear physics: Learn about extreme finite size scaling in real systems

M. Thorpe, MSU

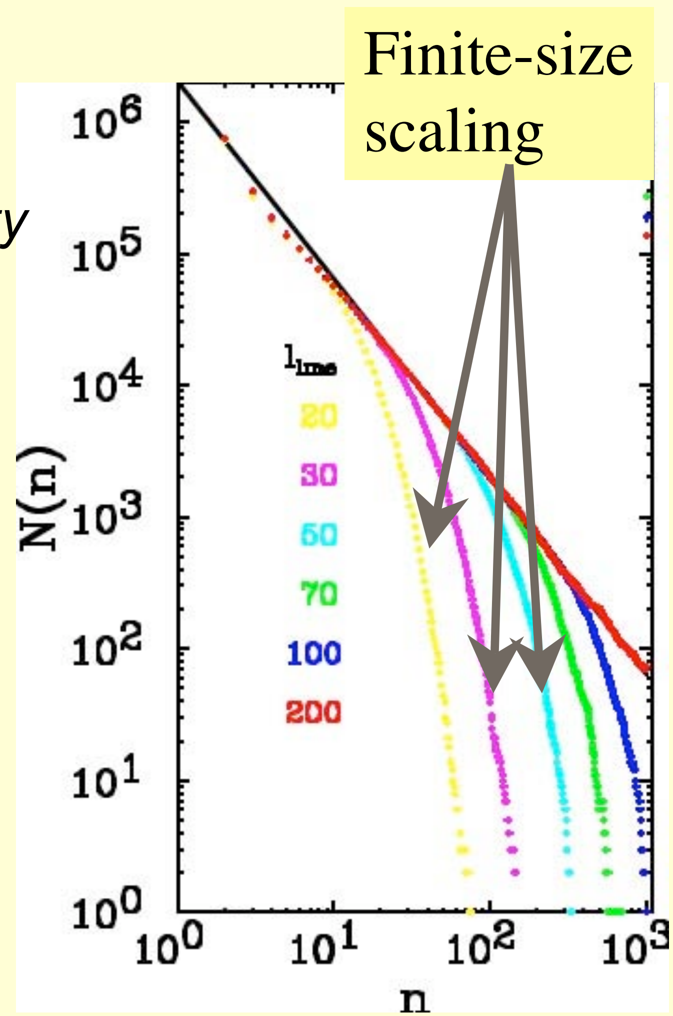


Self-Organized Criticality

- How to achieve scale-invariance?
  - *Vicinity of critical point: power-laws*
  - *Very careful tuning of control parameter(s) required*
- Another possibility: **SOC**
  - *Sequence of avalanches between metastable states*
  - *Continually driven to criticality*
  - *No external tuning required*
  - *Example: Bak's sand pile*

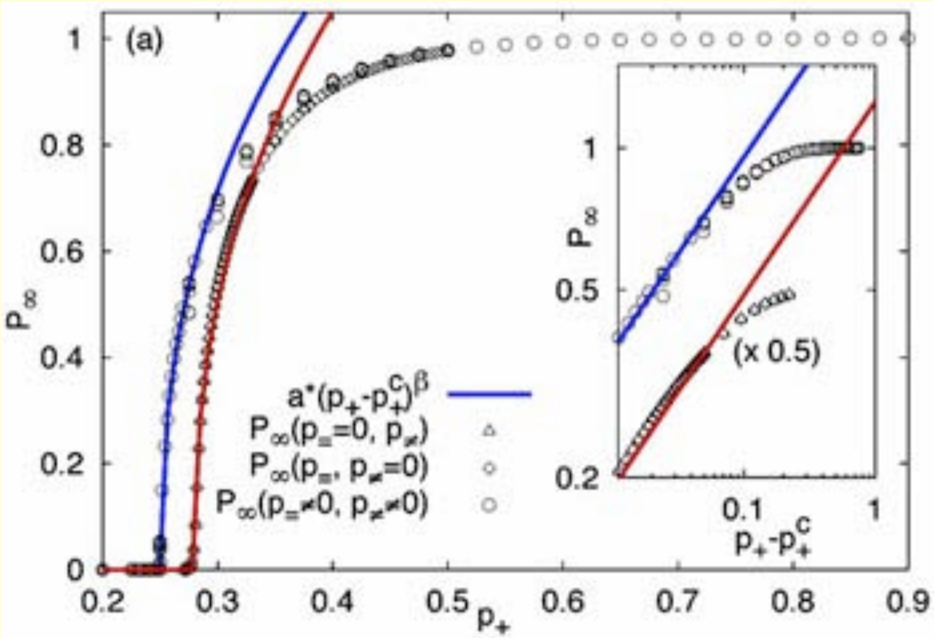


Held et al., PRL 65



Multi-Component Systems

- What happens when physically different components are in the system undergoing phase transition (protons+neutrons, different flavor quark&gluons, …)?
- Possible:
  - *Change of character of phase transition*
    Müller&Serot, PRC ***52***
  - *Shift in critical value of control parameter, same critical exponents*
    Harreis&WB, PRB ***62***



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Event-by-Event

- Near critical point, information on fluctuations is essential; averaging destroys it
- Promising candidates: E-by-E moment analyses

$$M_k(e) = \sum_i n_e(i)\, i^k.$$

e = event, $n_e(i)$ = # of times i is contained in e

- E-by-E for different observables can generate N-dimensional scatter plots
- Big question: How to sort events into classes?
- Natural choice: If you know control parameter, use it!
  (easy for theory, impossible for experiment)
- Closest choice: observable that is ~linear in control parameter.
- Attempt: use charged particle multiplicity, *m*.

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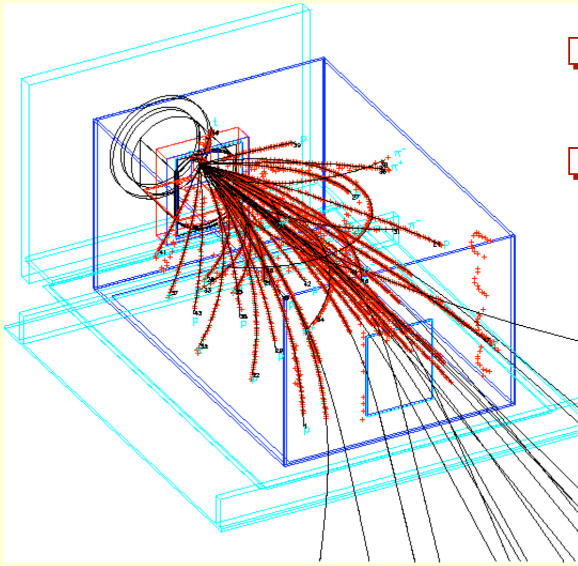
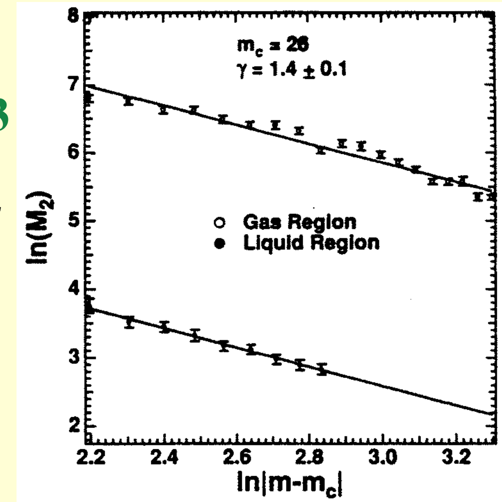
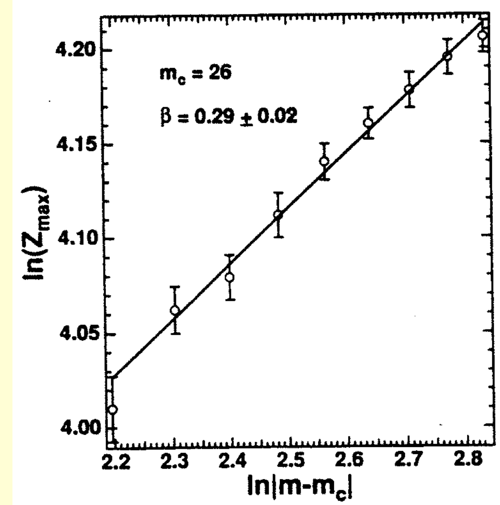
Determining Critical Exponents?



- EOS-TPC:
  Gilkes *et al.*, PRL **73**
- Complete reconstruction of events: all charges recovered





- Assume charged-particle multiplicity is proportional to control parameter
- Find critical value, m_c; extract critical exponents β and γ:
  $\gamma = 1.4$, $\beta = 0.29$
- Assuming validity of hyper-scaling: universality class of transition is completely determined

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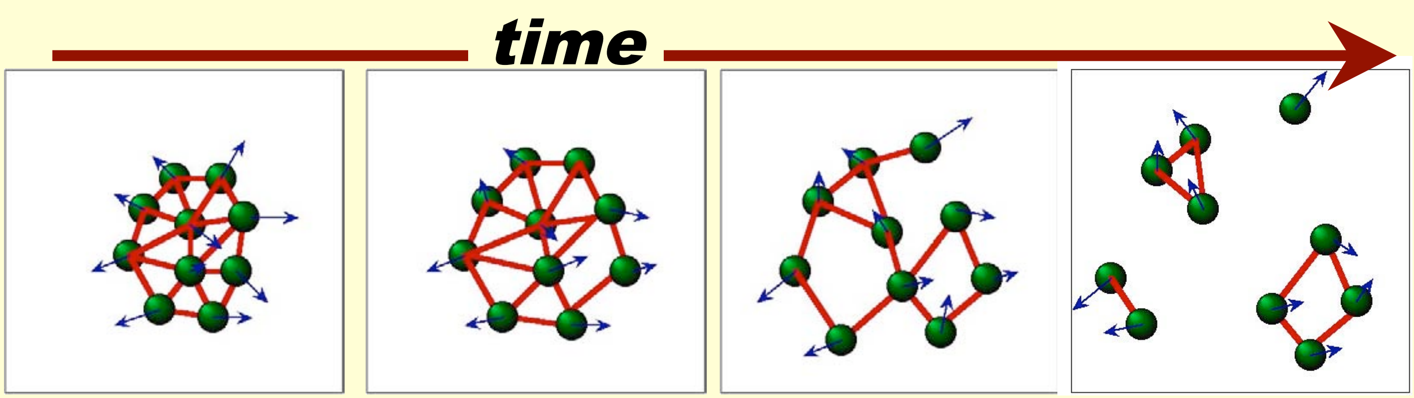
Percolation

WB *et al.*, PLB **150**, 53 (1985)
WB *et al.*, NPA **452**, 699 (1986)
X.Campi, JPA **19**, L917 (1986)
T. Biro *et al.*, NPA **459**, 692 (1986)
J. Nemeth *et al.*, ZPA **325**, 347 (1986)
…

- Short-range NN force: nucleons in contact with nearest neighbors
- Expansion (thermal, compression driven, dynamical, …)
- Bonds between nucleons rupture
- Remaining bonds bind nucleons into fragments
- One ***control parameter***: bond breaking probability



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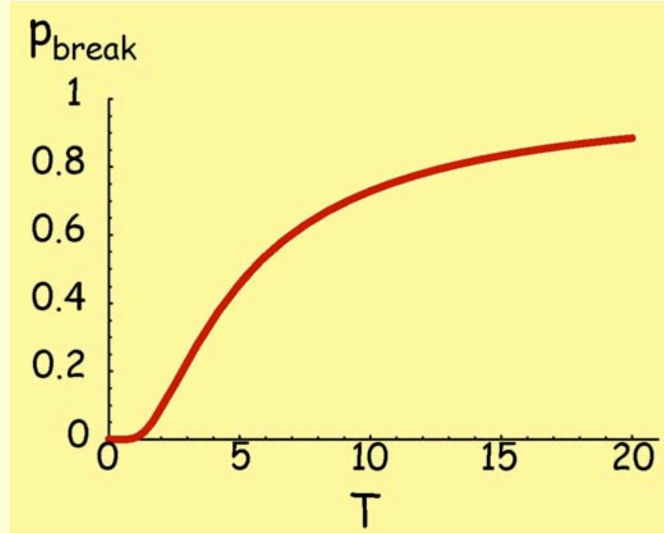
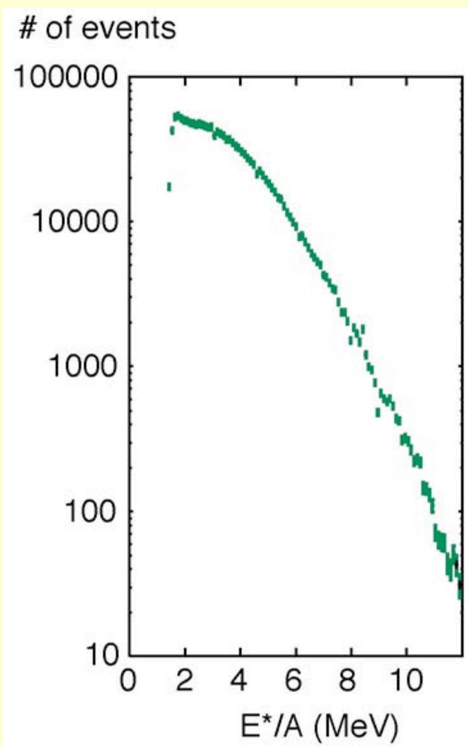
Breaking Probability

- **Determined by the excitation energy deposited**
- Infinite simple cubic lattice:
  - *3 bonds/nucleon*
  - *It takes 5.25 MeV to break a bond*





- p,π induced: Glauber theory
  - *p_{break} proportional to path length through matter*
- General relation between $p_{\rm break}$ and T:

$$p_{\rm break} = 1 - \frac{2}{\sqrt{\pi}}\,\Gamma\left[\frac{3}{2}, 0, \frac{B}{T}\right]$$

Γ = generalized incomplete gamma function, *B* = binding energy per nucleon
T. Li et al., PRL **70**
(generalization of Coniglio-Klein for Fermi systems)

- Obtain E^* or T from other model or directly from experiment

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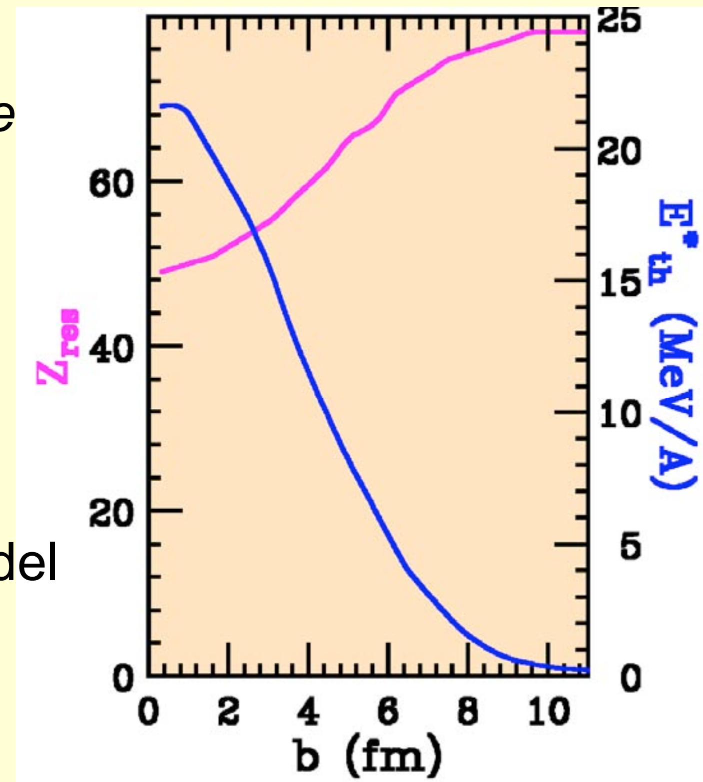
AA Collisions: Hybrid Model

- First stage: Intra-nuclear cascade (or other transport model)
  - *Produces distribution of residue sizes and E^**
  - *Convert E^* into temperature and percolation breaking probability*

$$p_{\text{break}} = 1 - \frac{2}{\sqrt{\pi}}\,\Gamma\left[\frac{3}{2}, 0, \frac{B}{T}\right]$$

- Second stage: Percolation model with lattice size = charge of residue
  - *Produces fragments*
- Total multiplicity = INC pre-equilibrium + percolation output

1 A GeV Au + C



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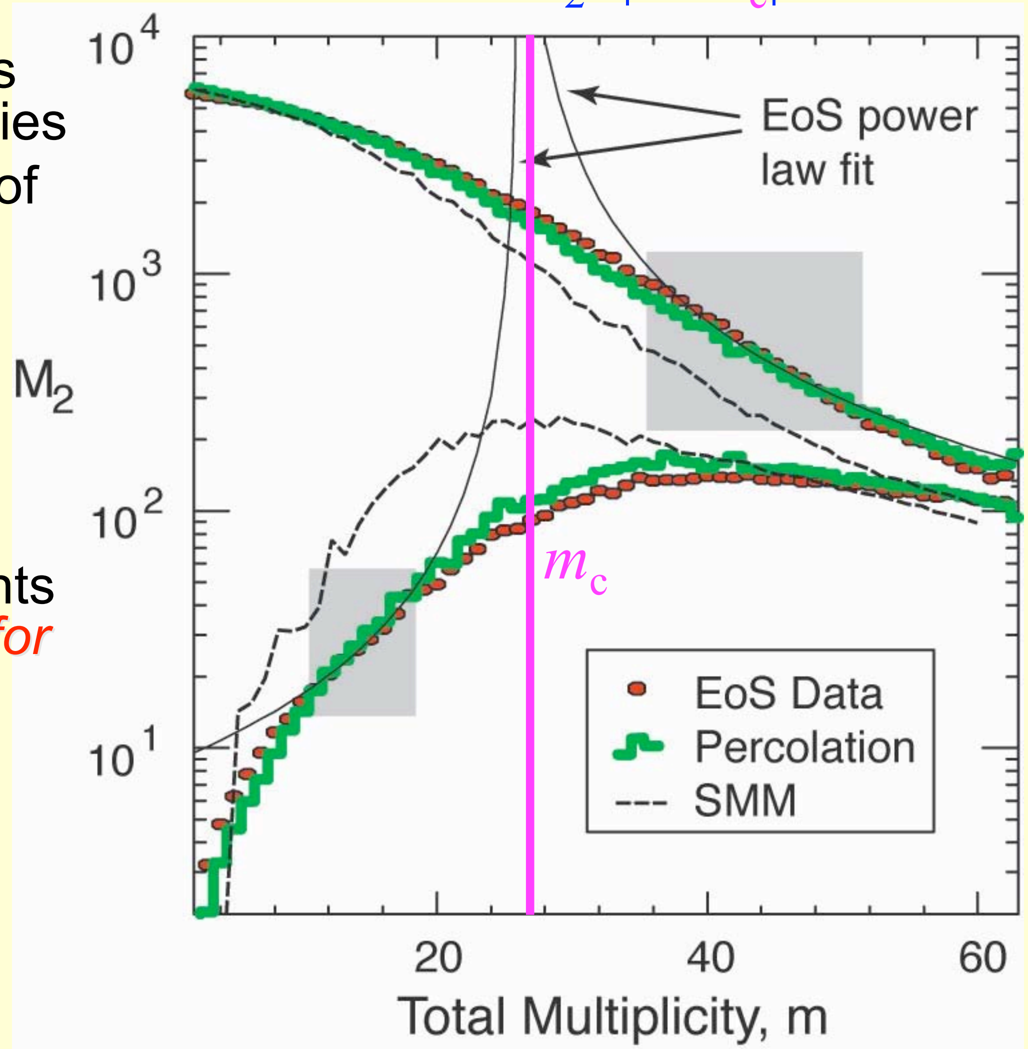
EoS Data / Percolation

- *1 A GeV Au + C*
- Data are integrated over all residue sizes and excitation energies
- Complete detection of all charges
- Data: red circles
- Percolation model: green histograms
- Percolation model contains critical events ⇒ *Strong evidence for continuous phase transition*
- But: $\gamma = 1.80$, $\beta = 0.41$

$M_2 \sim |m - m_c|^{-\gamma}$



WB & A.Botvina, PRC 52
WB & A.Botvina, PRC 55

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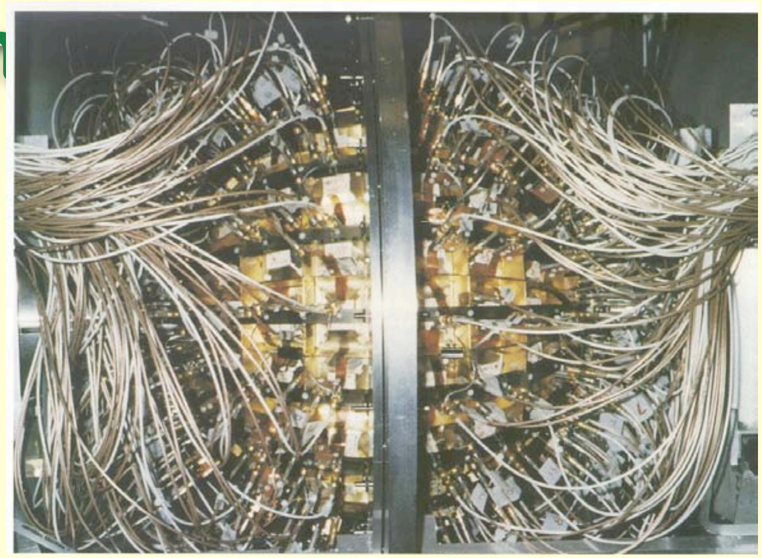
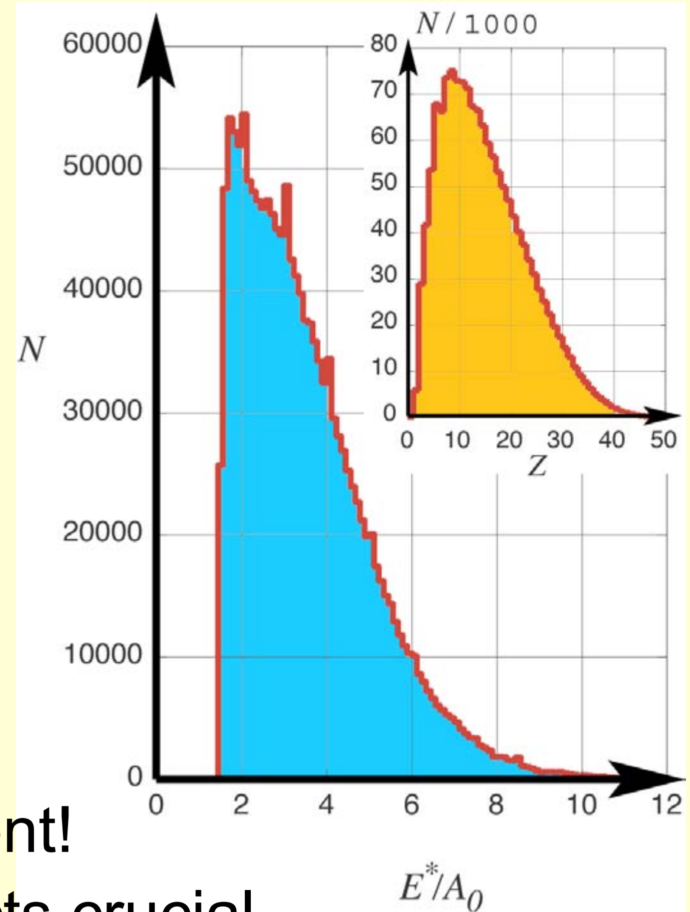
ISiS BNL Experiment



- 10.8 GeV p or π + Au
- Indiana Silicon Strip Array
- Experiment performed at AGS accelerator of Brookhaven National Laboratory

ISIS Data Analysis

- Marko Kleine Berkenbusch
- Collaboration w. Viola group



- Reaction:
  p, π+Au @AGS
- Very good statistics
  (~10^6 complete events)
- Philosophy: Don't deal with energy deposition models, but take this information from experiment!
- Detector acceptance effects crucial
  - *filtered calculations, instead of corrected data*
- **Parameter-free** calculations

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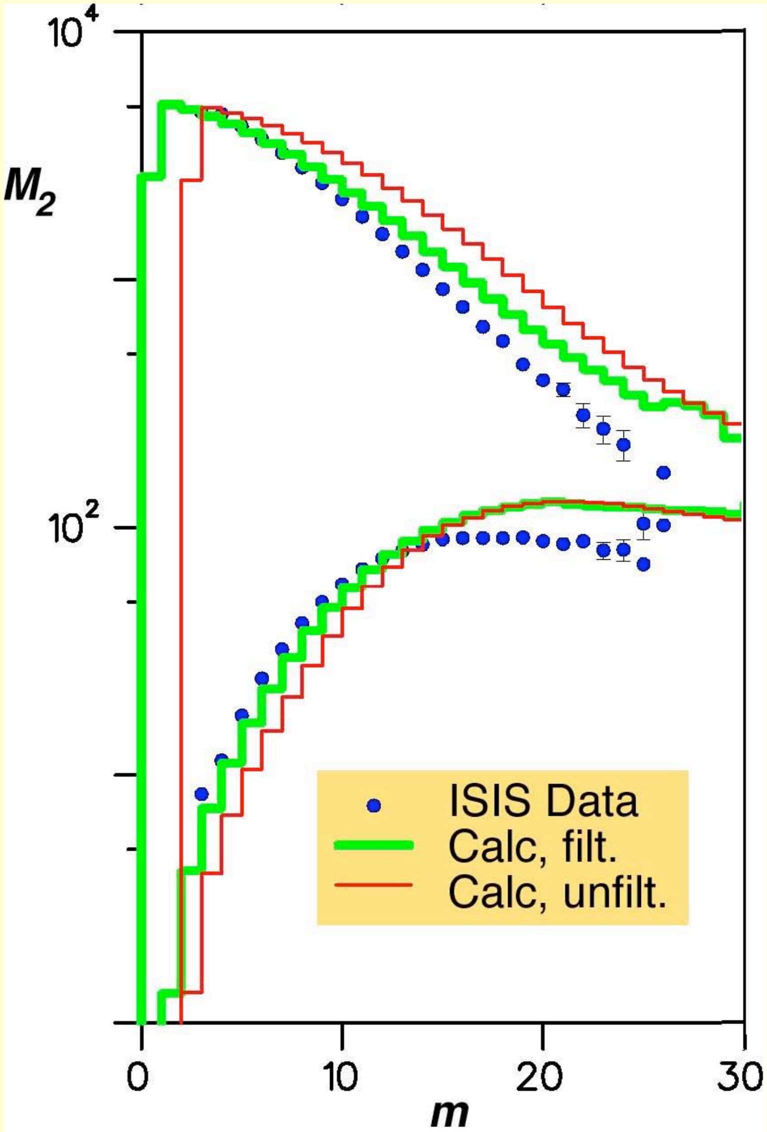
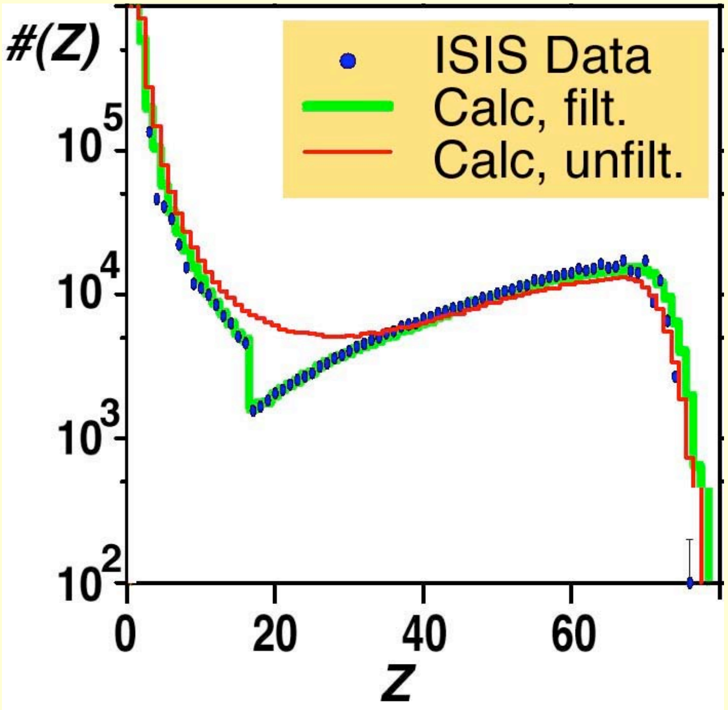
Comparison





- Charge yield spectrum
- Second moments
- **Very good agreement** between theory and data
  - *Filter very important*
  - *Sequential decay corrections huge*

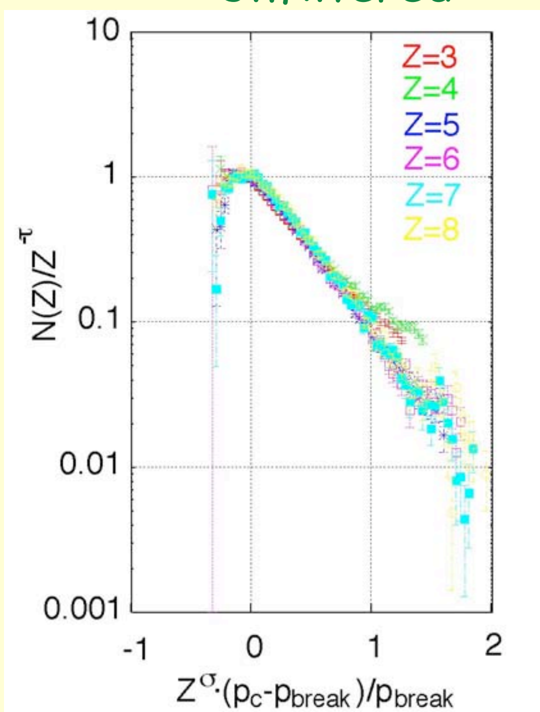
Scaling Analysis

- Idea (Elliott et al.): If data follow scaling function

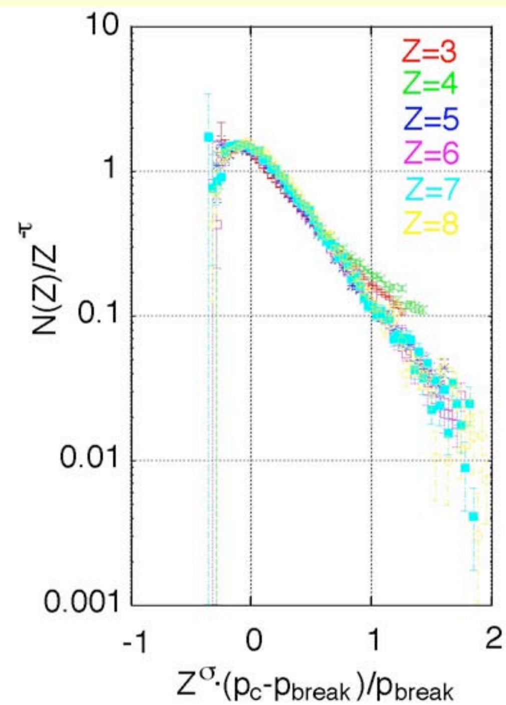
$$N(Z,T) = Z^{-\tau} f\left[\frac{T - T_c}{T_c} Z^{\sigma}\right]$$

with $f(0) = 1$ (think “exponential”), then we can use scaling plot to see if data cross the point [0,1] -> critical events

Unfiltered



Filtered



- Note:
- Critical events present, $p > p_c$
- Critical value of p was corrected for finite size of system

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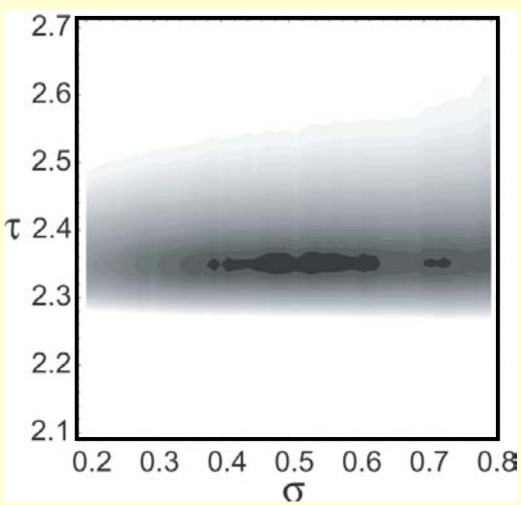
Scaling of ISIS Data

- Most important: critical region and explosive events probed in experiment
- Possibility to narrow window of critical parameters
  - *τ: vertical dispersion*
  - *σ: horizontal dispersion*
  - *T_c: horizontal shift*
- χ^2 Analysis to find critical exponents and temperature



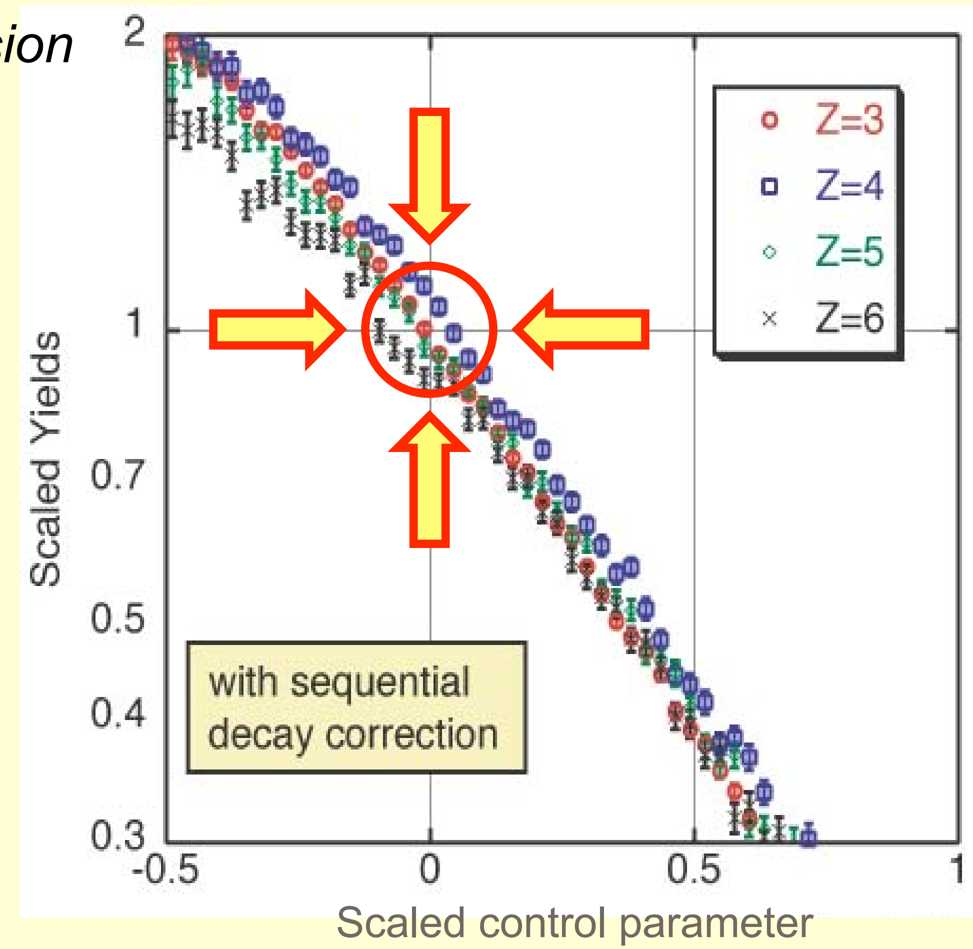
- Result:

  $\sigma = 0.5$ +- 0.1

  $\tau = 2.35$ +- 0.05

  $T_c = 8.3$ +- 0.2 MeV

M. Kleine Berkenbusch *et al.,* PRL **88**



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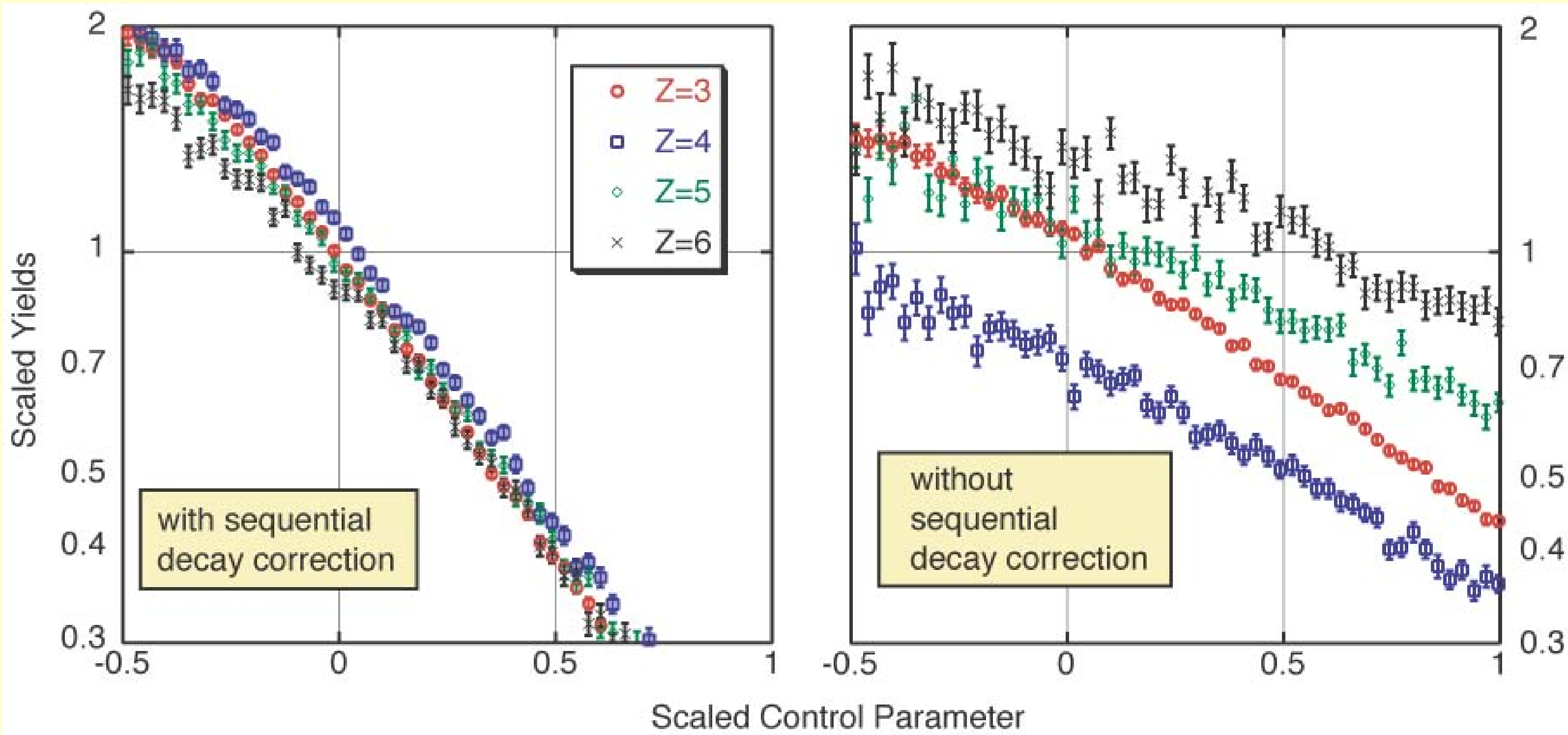
Scaling of ISIS Data (2)

- Note: This only works because of very careful correction for sequential decays!
- Best-fit scenarios for both cases:
  - *Scaling collapse only when sequential decay correction is performed*
  - *Technique fails without it*



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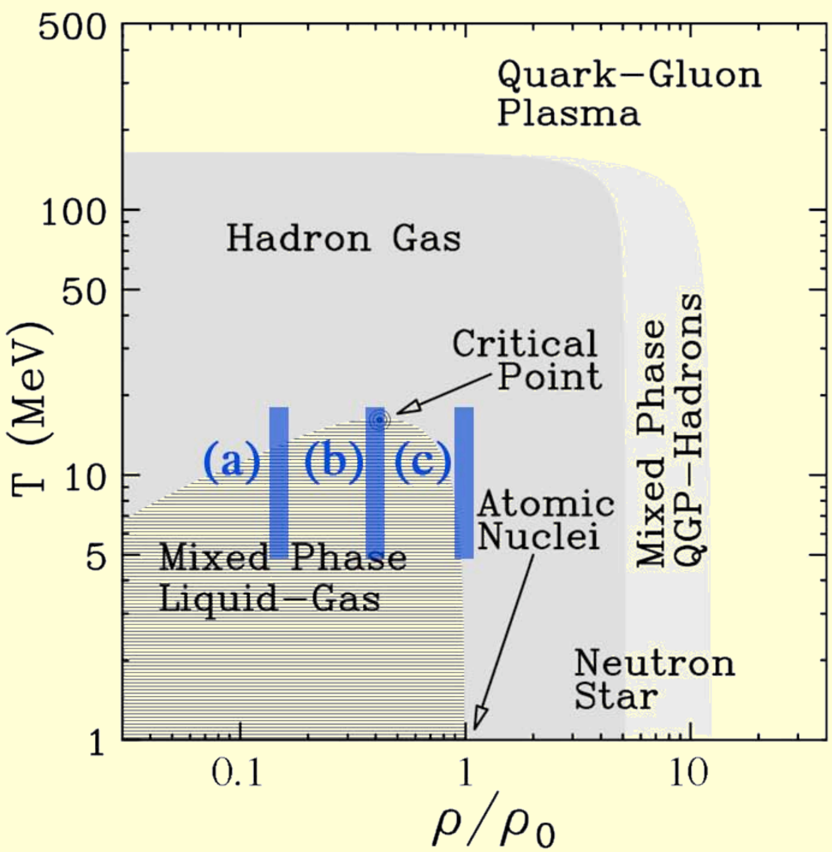
Freeze-Out Density

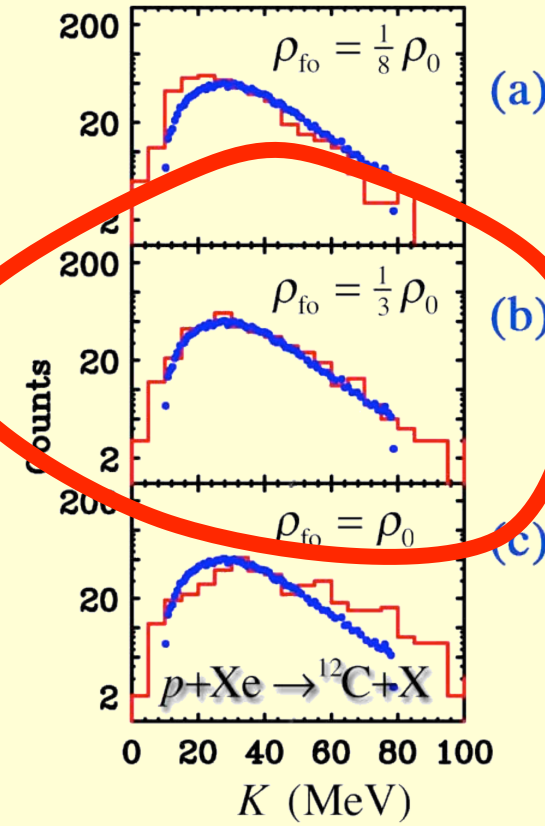
$$p_b = 1 - \frac{2}{\sqrt{\pi}} \Gamma(\tfrac{3}{2}, 0, B/T)$$

- Percolation model only depends on breaking probability, which can be mapped into a temperature.
- **Q**: How to map a 2-dimensional phase diagram?
- **A**: Density related to fragment energy spectra; Coulomb many-body expansion of pre-fragments

WB, Alleman, Pratt, AIP conf.proc.884, 327 (2007)
WB, Nucl.Phys. A**787**, 595c (2007)

$$\rho_c = (0.35 \pm 0.1)\rho_0$$





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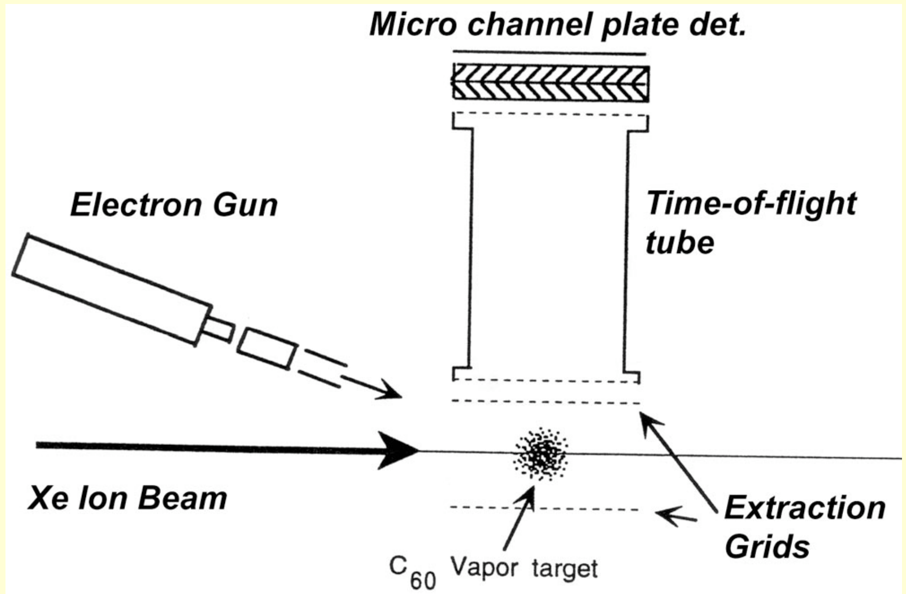
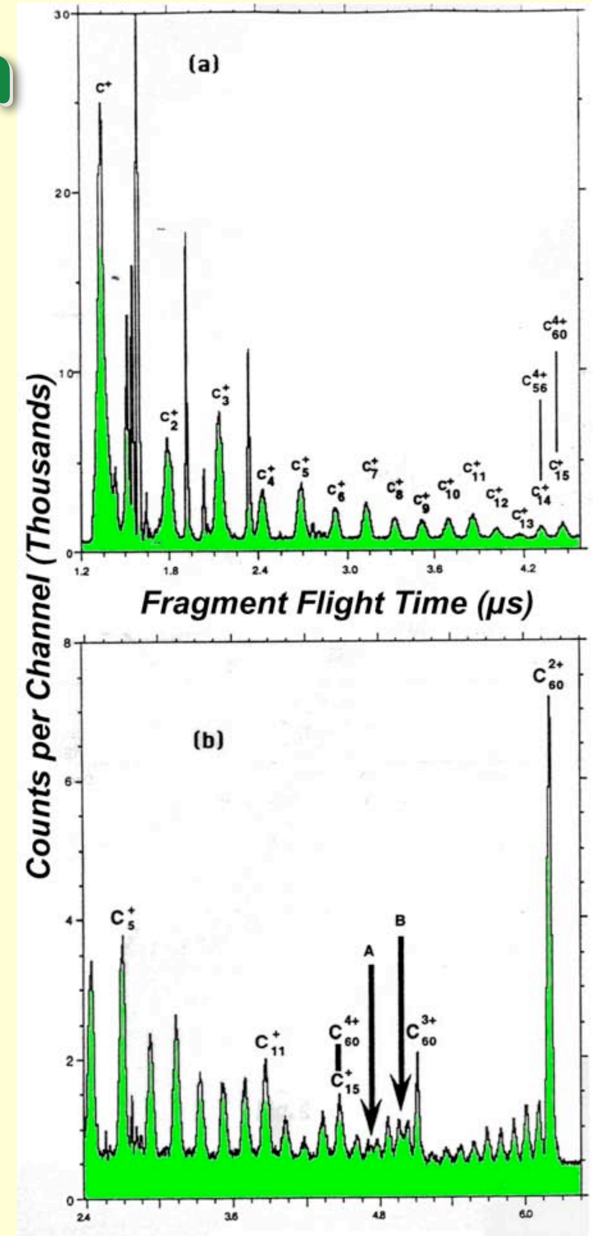
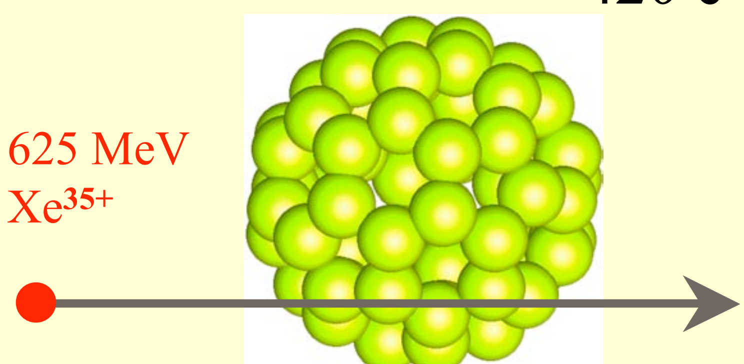
Buckyball Fragmentation





Binding energy of C_{60}:
420 eV



Cheng et al., PRA 54

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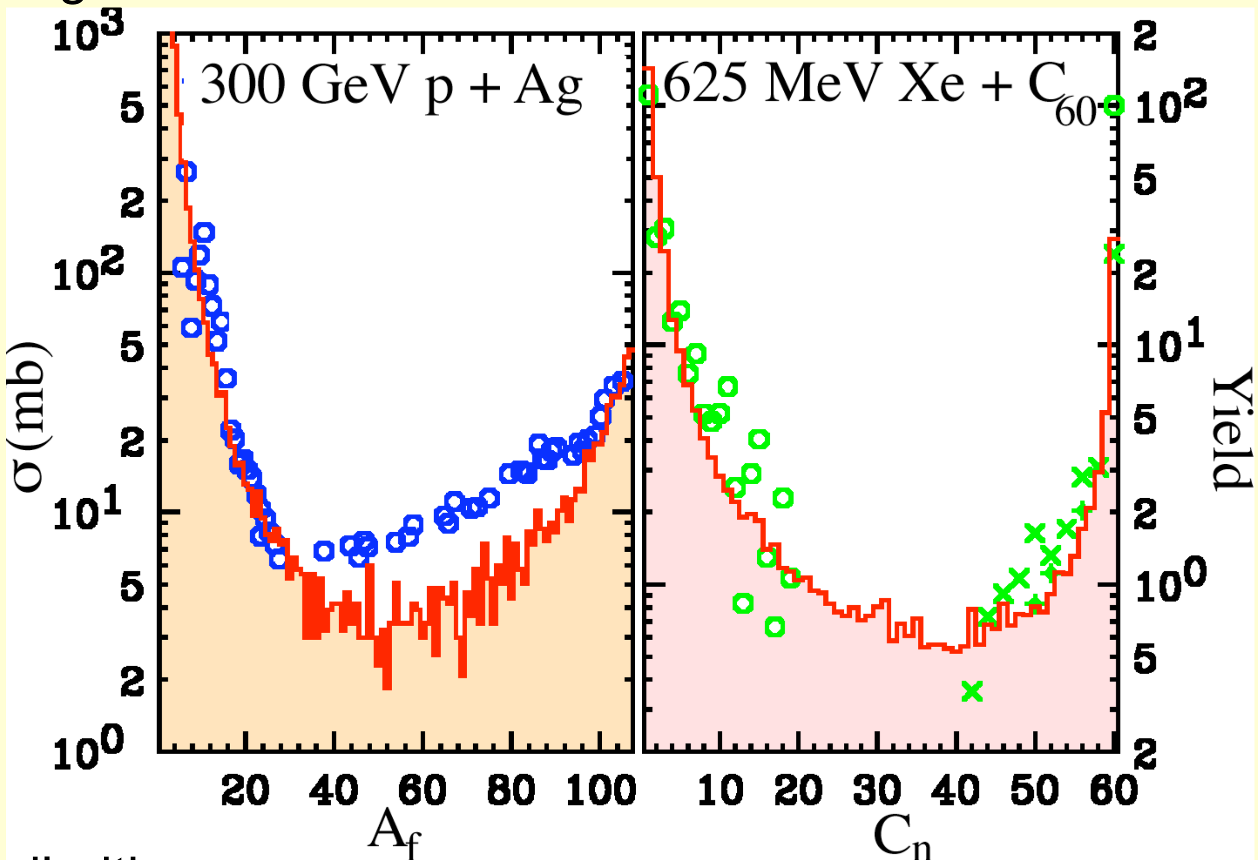
Cross-Disciplinary Comparison

Data:
Bujak et al., PRC 32
LeBrun et al., PRL 72
Calc.:
WB, PRC 38
Cheng et al., PRA 54

- Left: Nuclear Multifragmentation
- Right: Buckyball Fragmentation
- Histograms: Percolation Models



- Similarities:
  - *U - shape (b-integration)*
  - *Power-law for imf's (1.3 vs. 2.6)*
  - *Binding energy effects provide fine structure*

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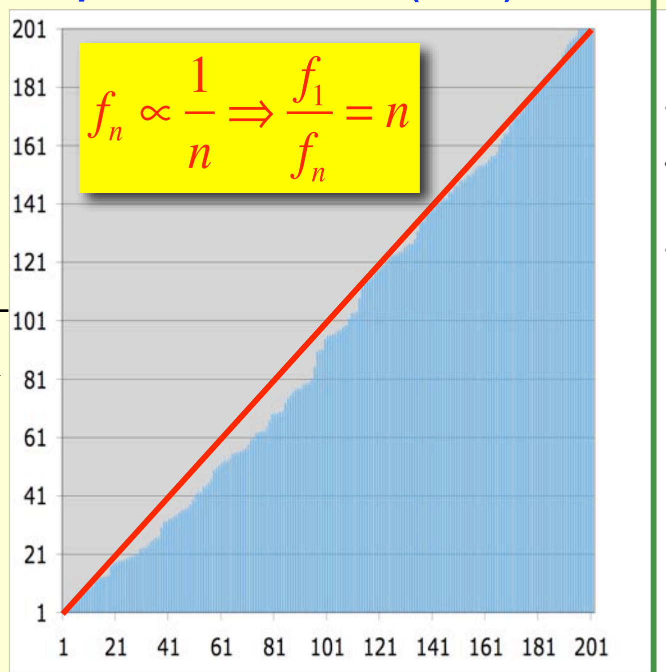
Zipf's Law

- Count number of words in a book (in English) and order the words by their frequency of appearance
- Find that the most frequent word appears twice as often as next most popular word, three times as often as 3rd most popular, ...

G. K. Zipf, *Human Behavior and the Principle of Least Effort* (1949)



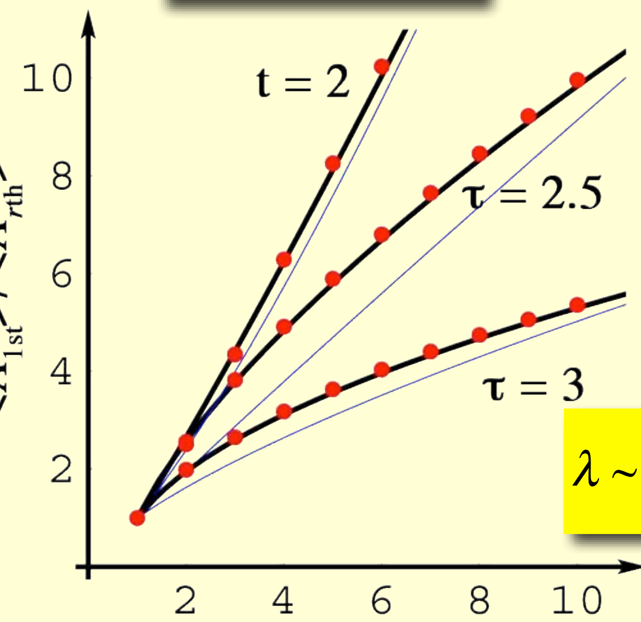
$$f_n \propto \frac{1}{n} \Rightarrow \frac{f_1}{f_n} = n$$

$$1.4\,\frac{f_1}{f_n}$$

n

- Nuclear fragmentation: size of clusters vs. their rank follows generalized Zipf-Mandelbrot law, with power law exponent determined by critical exponent.

$$\langle A_{r\text{th}} \rangle = \frac{c}{(r+k)^{\lambda}}$$



$<A_{1\text{st}}>/<A_{r\text{th}}>$

r

$$\lambda \sim \frac{1}{\tau - 1}$$

K. Paech, W. Bauer, S. Pratt, PRC 76, 2007, arXiv:0706.0913v1

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Zipf's Law: Probabilities (1)

- Probability that cluster of size A is the largest one = probability that at least one cluster of size A is present times probability that there are 0 clusters of size >A

$$P_{1st}(A) = p_{\geq 1}(A) \cdot p_0(>A)$$
$$= [1 - p_0(A)] \cdot p_0(>A)$$

- $N(A)$ = average yield of size A: $N(A) = aA^{-\tau}$
- $N(>A)$ = average yield of size >A: (V = event size)

$$N(>A) = \sum_{i=A+1}^{V} N(i) = \sum_{i=A+1}^{V} a i^{-\tau} = a\zeta(\tau, 1+A) - a\zeta(\tau, 1+V)$$

- Normalization constant a from condition: $\sum_{A=1}^{V} A \cdot N(A) = V$

$$a = V \Big/ \sum_{A=1}^{V} A^{1-\tau} = V / H_V^{(1-\tau)}$$

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Zipf's Law: Probabilities (2)

- Use Poisson statistics for individual probabilities:

$$p_n(i) = \frac{\langle N(i)\rangle^n e^{-\langle N(i)\rangle}}{n!}$$

$$p_0(i) = e^{-\langle N(i)\rangle};\ p_1(i) = \langle N(i)\rangle\, p_0(i);\ p_2(i) = \tfrac{1}{2}\langle N(i)\rangle\, p_1(i)\ldots$$

- Put it all together:

$$\begin{aligned} P_{1st}(A) &= [1 - p_0(A)] \cdot p_0(>A) \\ &= [1 - e^{-N(A)}] \cdot e^{-[a\zeta(\tau,1+A) - a\zeta(\tau,1+V)]} \end{aligned}$$

- Average size of biggest cluster

$$\langle A_{1st}\rangle = \sum_{A=1}^{V} A \cdot P_{1st}(A)$$

(Exact expression!)

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Zipf's Law: Probabilities (3)

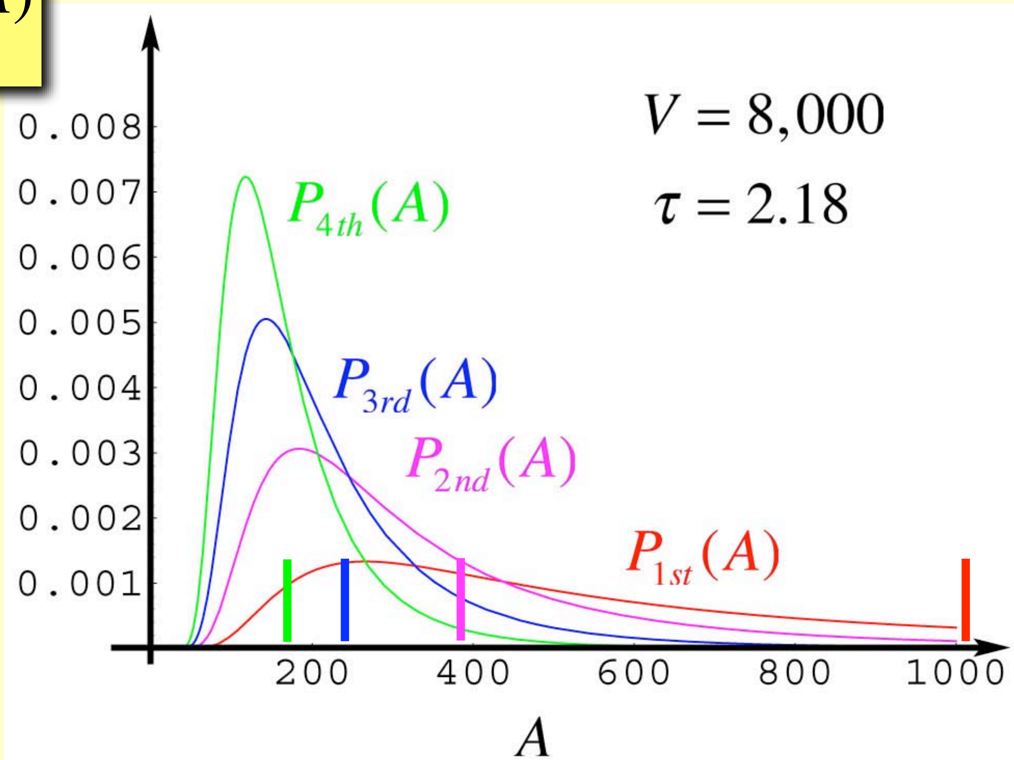
- Probability for given A to be 2nd biggest cluster:

$$P_{2nd}(A) = p_{\geq 2}(A)\cdot p_0(>A) + p_{\geq 1}(A)\cdot p_1(>A)$$
$$= [1 - p_0(A) - p_1(A)]\cdot p_0(>A) + [1 - p_0(A)]\cdot p_1(>A)$$

- Average size of 2nd biggest cluster:

$$\langle A_{2nd}\rangle = \sum_{A=1}^{V} A\cdot P_{2nd}(A)$$

- And so on …
- Recursion relations!



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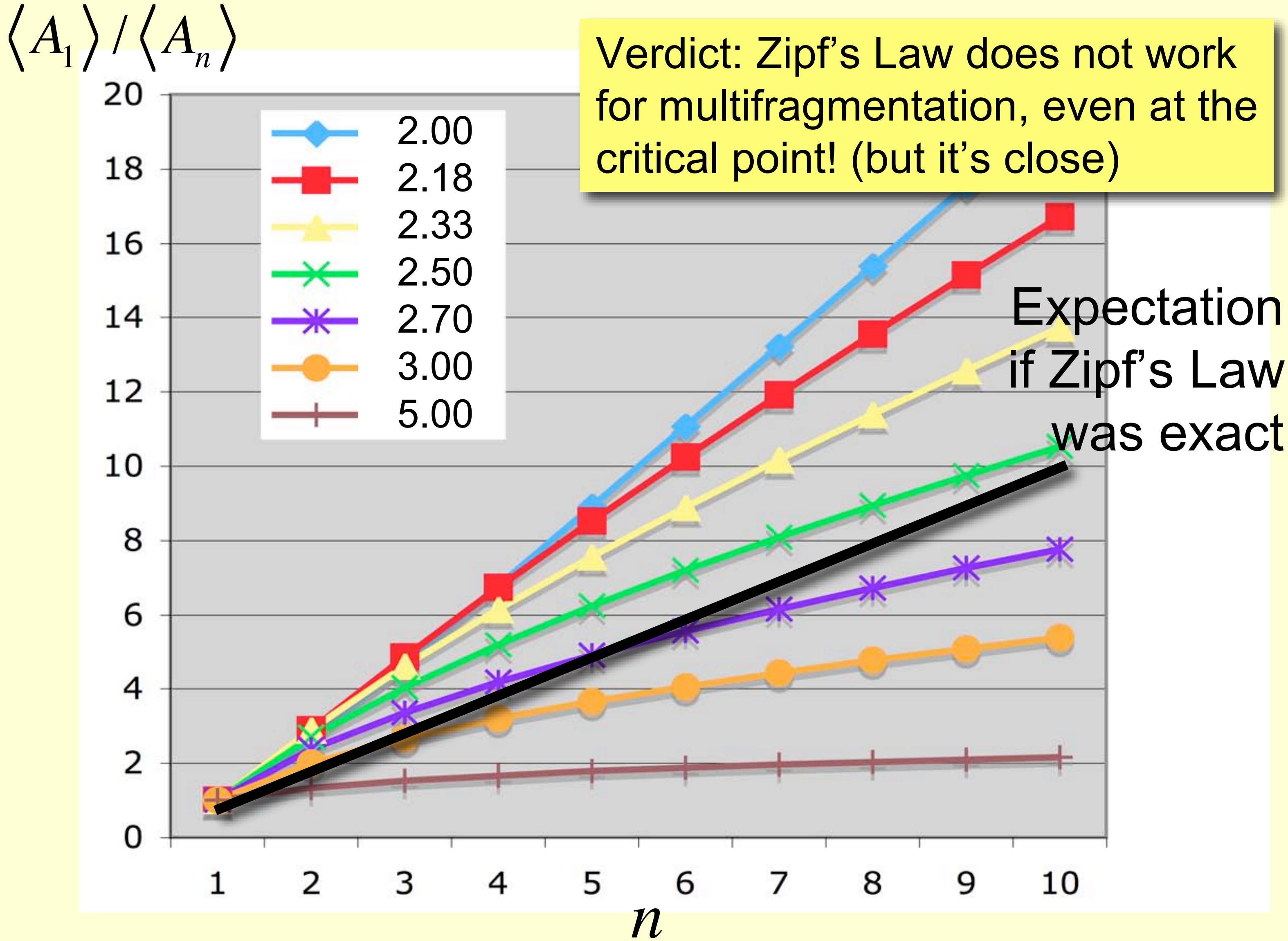
Zipf's Law: τ-dependence



Verdict: Zipf's Law does not work for multifragmentation, even at the critical point! (but it's close)

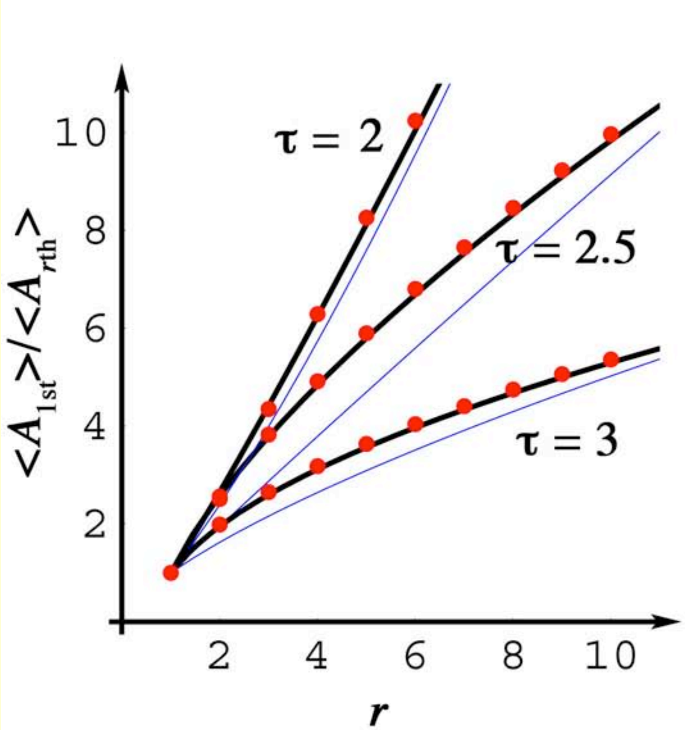
Resulting distributions: Zipf-Mandelbrot

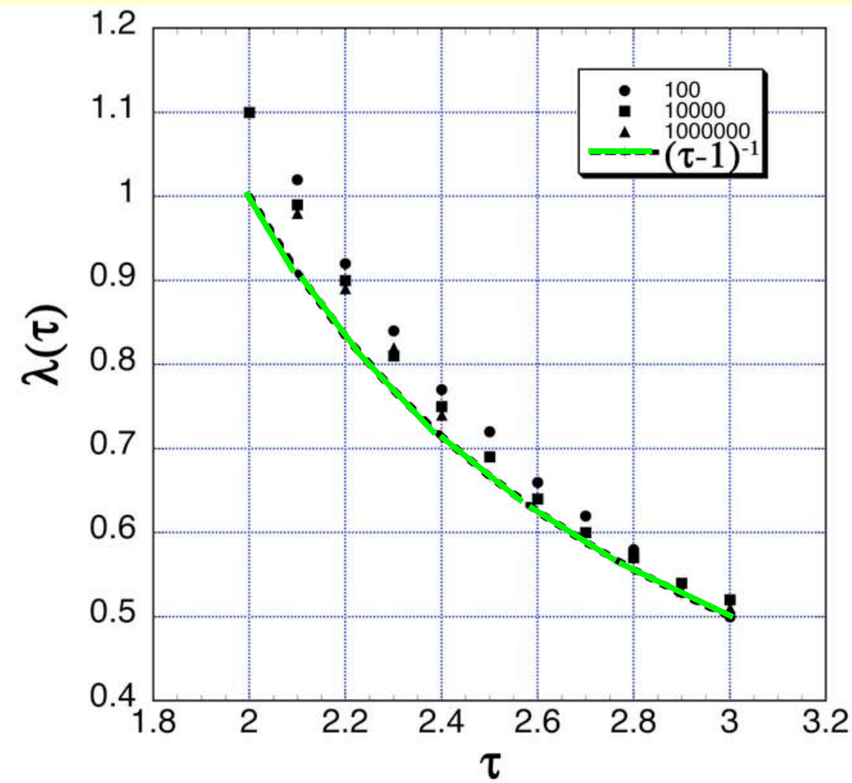
Zipf-Mandelbrot

- Limiting distributions for cluster size vs. rank

$$\langle A_{r\text{th}} \rangle = \frac{c}{(r+k)^{\lambda}}$$ with exponent $$\lambda \sim \frac{1}{\tau - 1}$$

- Proof for infinite system in continuum limit with τ=2: Paech, WB, Pratt, PRC 76, 2007





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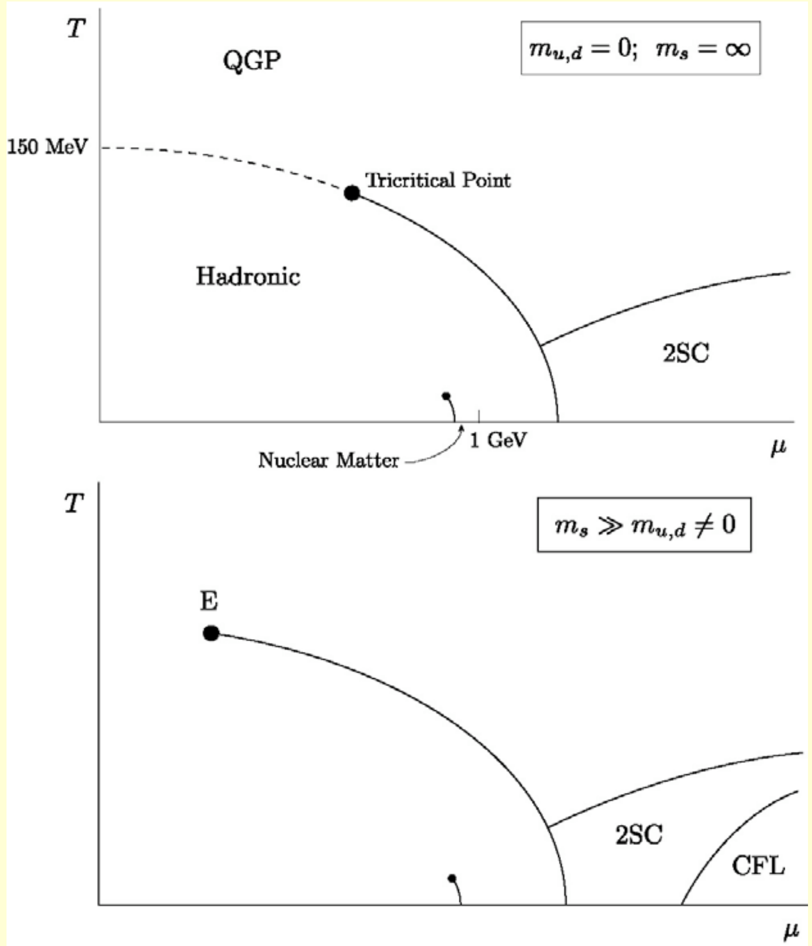
How to find the QCD Critical Point

- There are no large fragments!
- What clusters? What fluctuates?
- What is the order parameter that can be measured experimentally?
- What should CBM look for?



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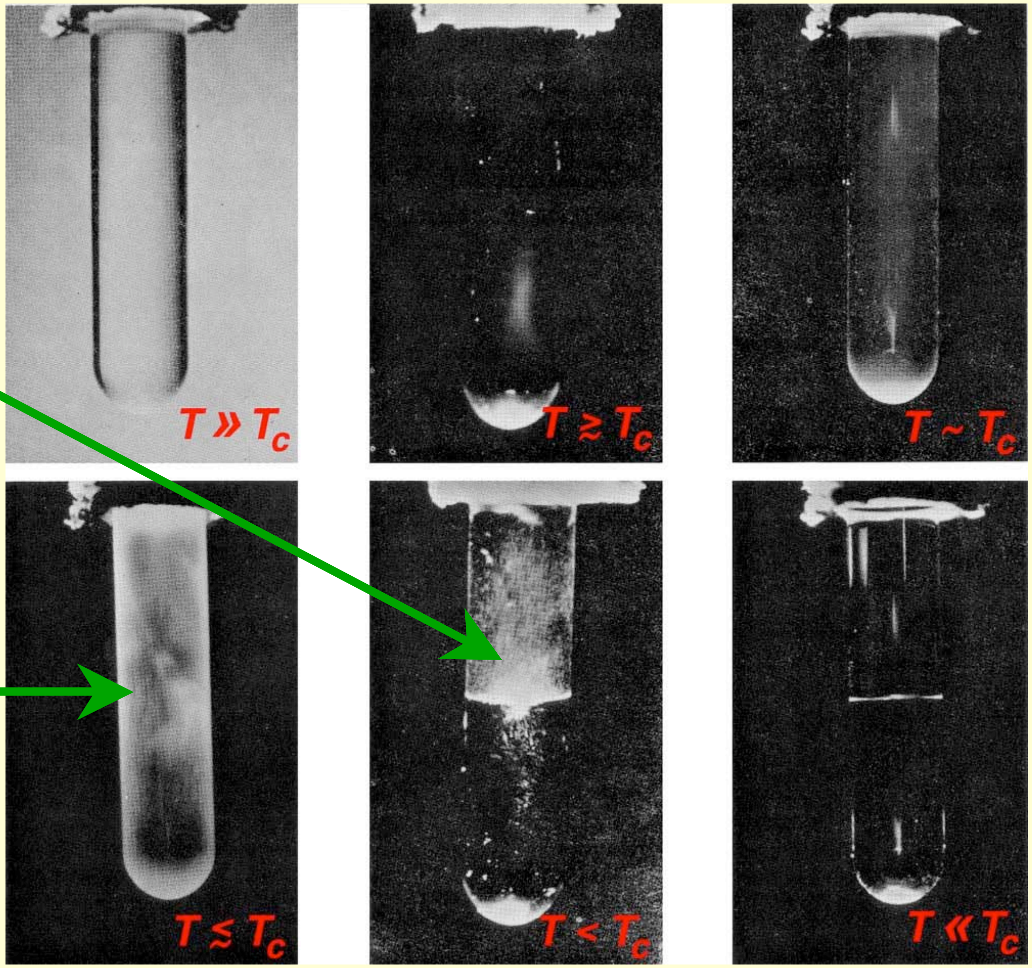
Liquid at the Critical Point

Cyclohexane-Aniline

Liquid-vapor coexistence

Critical Opalescence: Light scattering off fluctuations



Ferrell (1968)
Stanley (1971)

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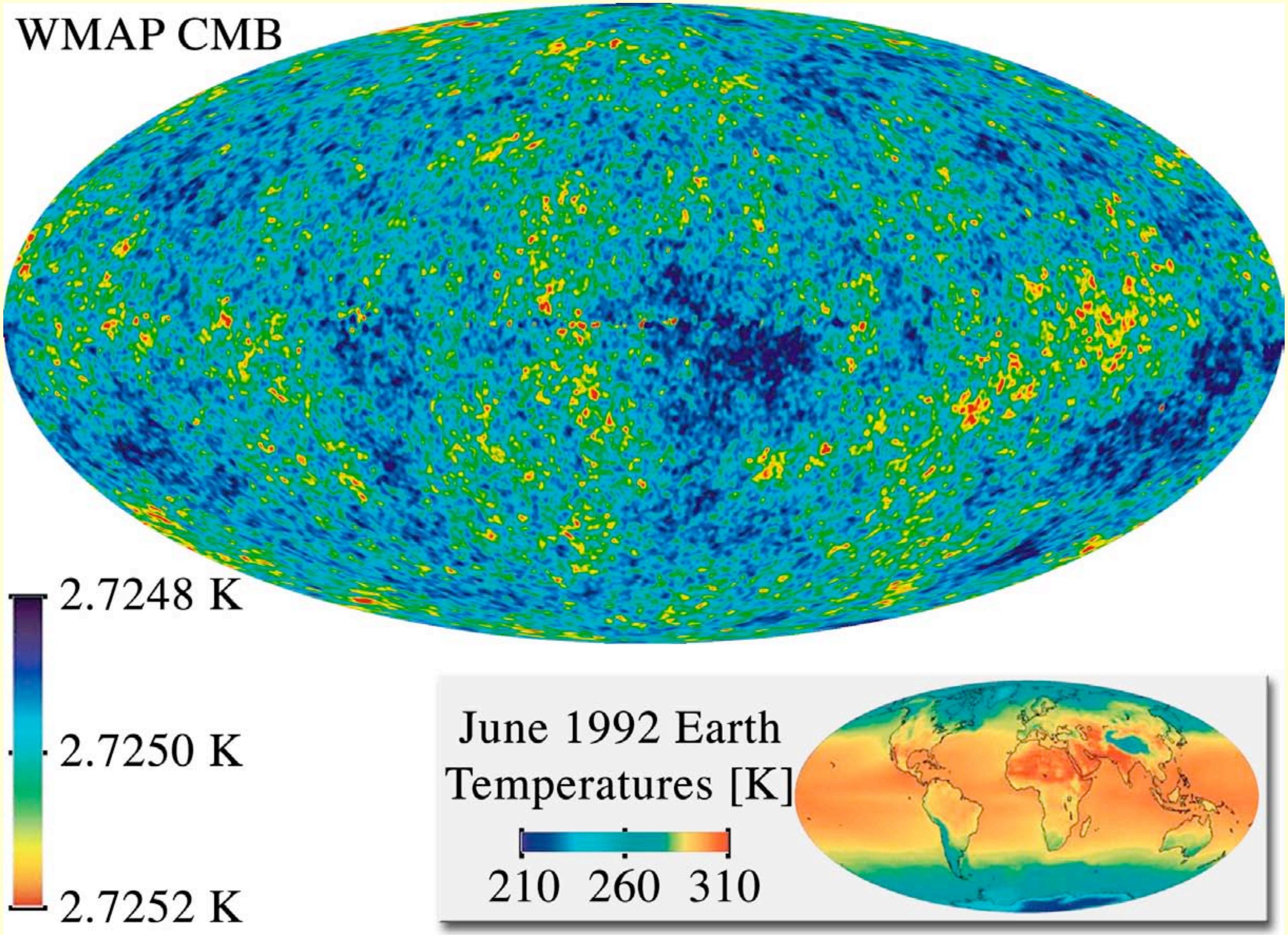
Fluctuations



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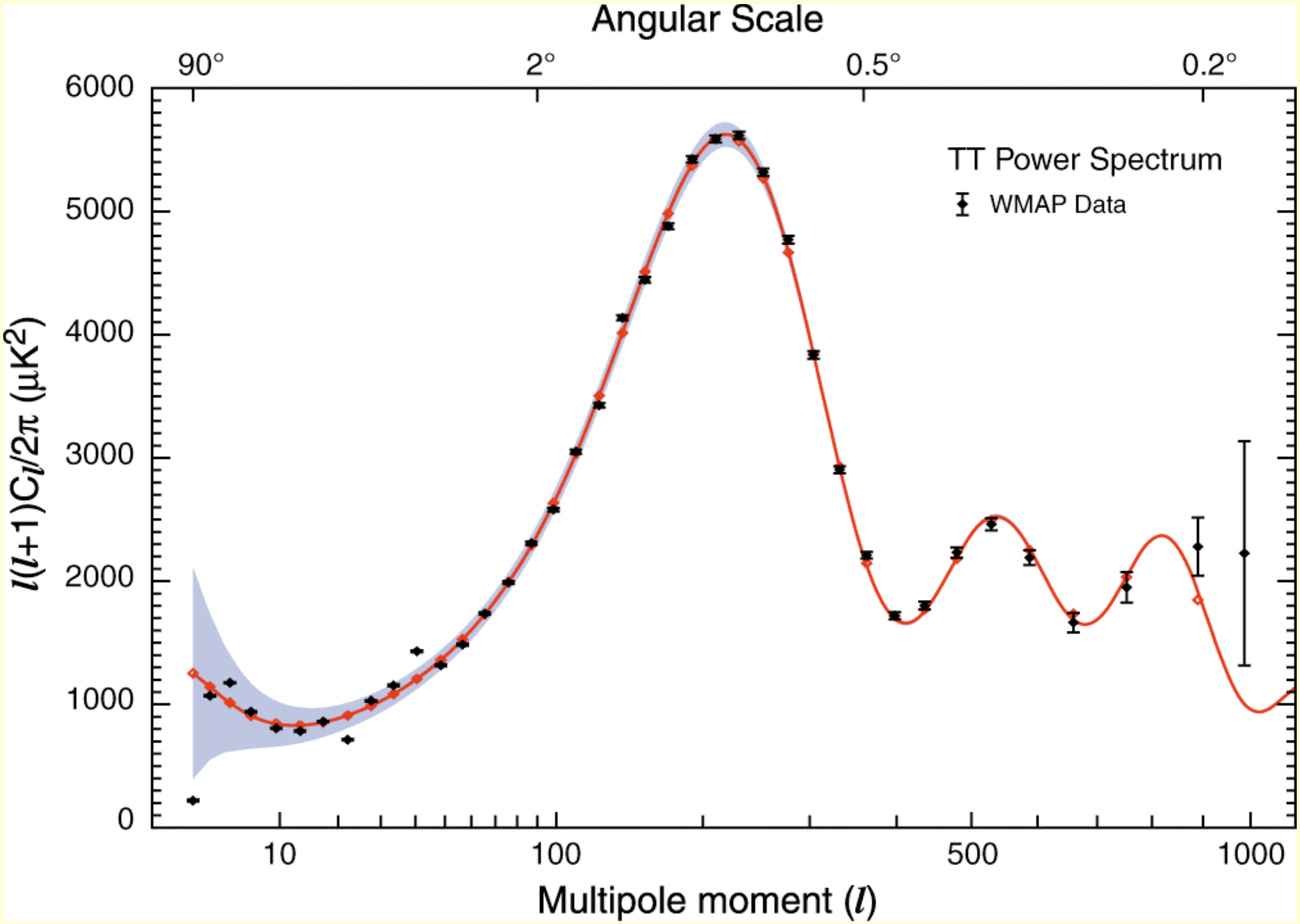
Multipole Analysis



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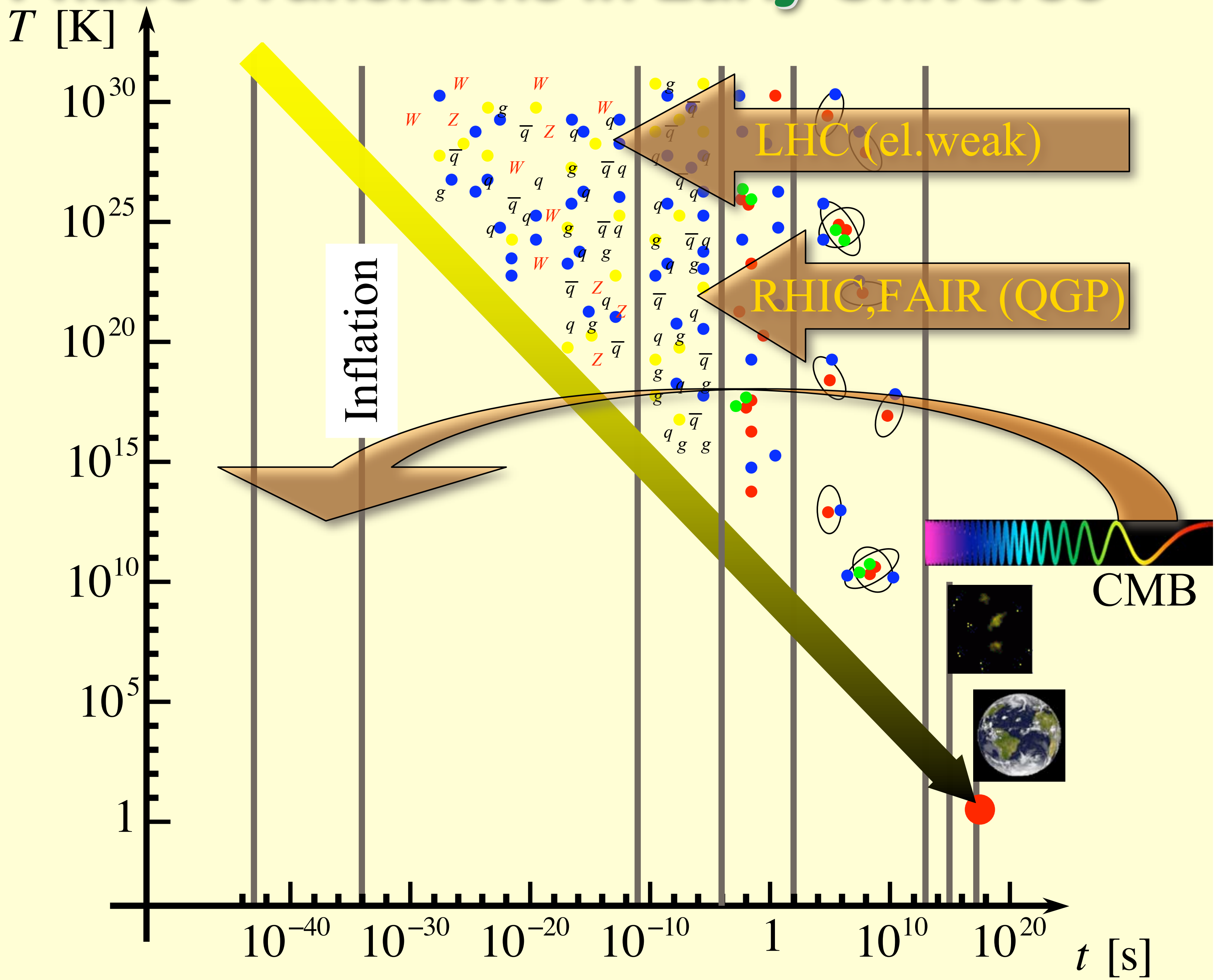
Phase Transitions in Early Universe



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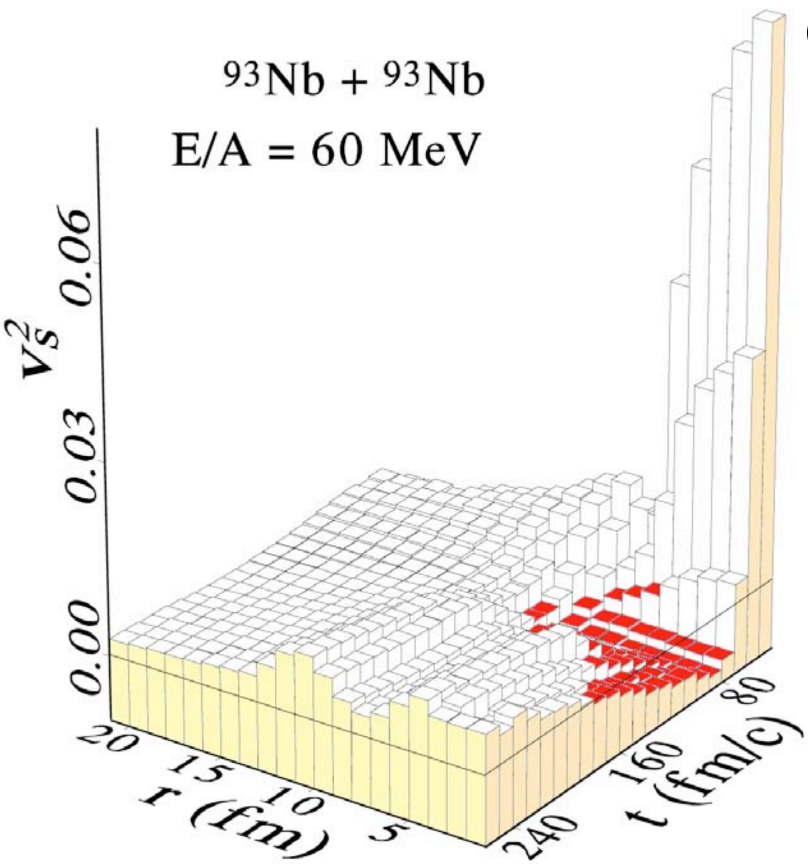
The "CMB" of CBM

- Photons emitted from early collision stages scatter off the fluctuations
  - *critical opalescence*
- Similar effect for pions
  - *"critical pion opacity"*
- Cluster analysis in momentum space for pions and photons
- FAIR chance that the signal of the critical point survives the later stages of final state interaction
- Finite size constraints
  - *Do not expect a sharp peak!*
  - *A bump is all that you will get at best*
  - *Unambiguous experimental signals are hard to come by*
  - *Lots of modeling needed to interpret results*

Dynamics

- Thermal equilibrium assumptions not (always) valid
- Need transport theory
- Various event class averages (event vs. thermal!)
- Connections to underlying phase diagram poorly understood



- Transient formation of non-compact structures
  - *Sheet instabilities* *Moretto et al., PRL 69*
  - *Bubble and ring formation* *WB, Schulz, Bertsch, PRL 69*
  - *Imaginary sound velocity causes exponential growth in fluctuations; non-equilibrium in origin*
  - *Similar effect now postulated for RHIC collisions* (*Pratt 2008*)

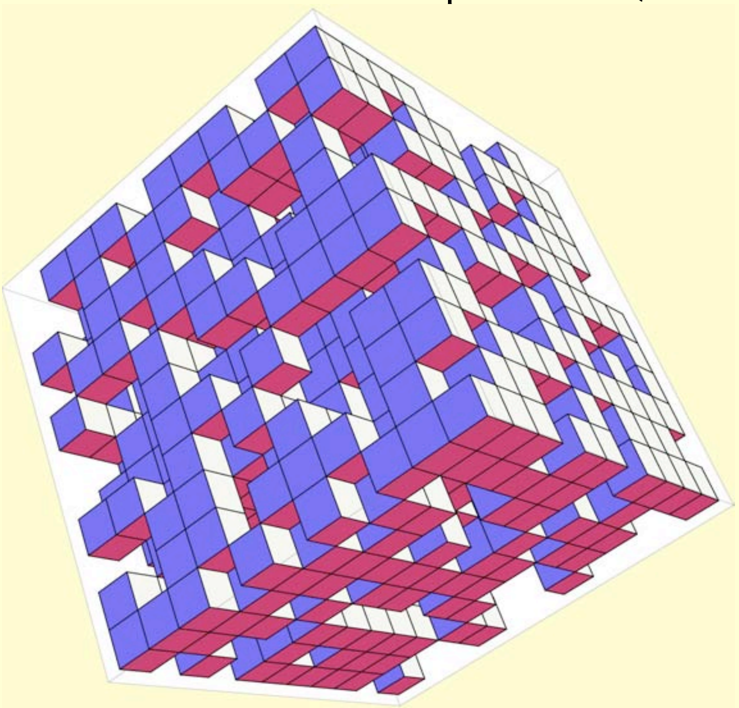
Critical Slowing Down

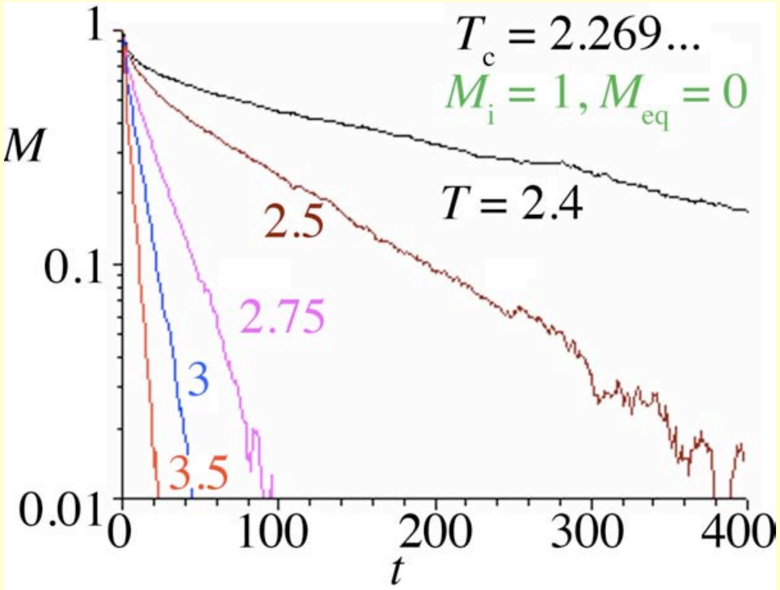
- Near critical point, $|T/T_c| \ll 1$, it takes longer and longer to re-establish equilibrium after changing the temperature
- Example: Ising Model,

$$M(t) \sim \exp(-\,t/t_r),$$

with

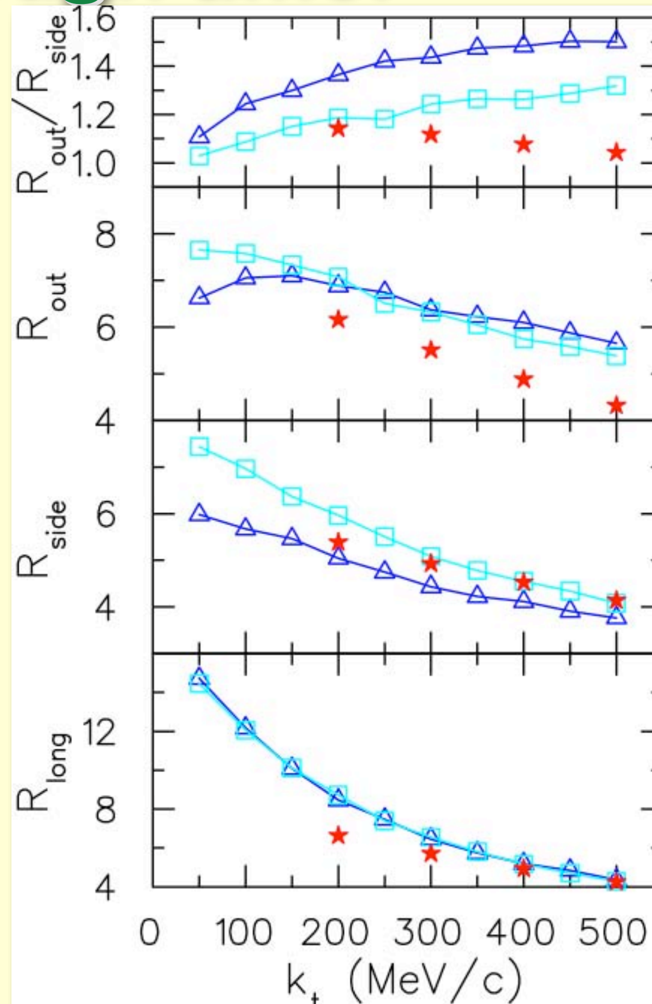
$$t_r = 4.5\,(T-T_c)^{-1.85}, \text{ for } T > T_c.$$





… but there is not enough time!

- HBT puzzle
- Theoretical expectation
  - *Change of # of degrees of freedom in transition from quarks and gluons to hadrons*
  - *Large time delay*
  - *Expect $R_{out} >> R_{side}$*
- Not seen by experiment! (wait for talk by Scott Pratt)



- Equilibrium thermodynamic phase transition may not be possible
  - *… but percolation-type transition not excluded!*

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Conclusions

- Common threads in phase transitions of molecules, nuclei, and hadronic matter
- Non-equilibrium effects make extraction of EoS information hard
- View of multifragmentation as a **critical phenomenon** is on solid footing
- Critical slowing down not important, because it is a non-equilibrium phase transition
- Finite-size corrections can be dealt with rather effectively; opportunity for us to contribute to larger science community
- 2nd critical point has all of the above qualifiers as well, but there is a fighting chance for success
- CBM experiment and RHIC low energy scan needs to concentrate on measuring fluctuations in distributions of baryons, pions, (early) photons

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