
Universality class **of the chiral phase transition** **from lattice simulations**

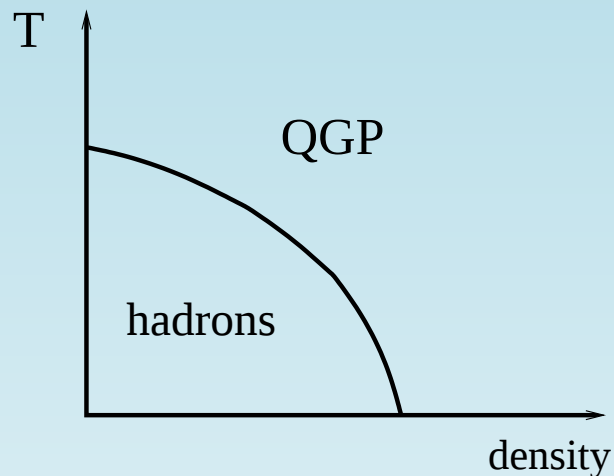
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Abstract

It is clear that the order of the deconfining phase transition in QCD at nonzero temperature and density (with realistic quark masses) is of great experimental interest. The importance of the transition at **zero baryon density** with only **two (degenerate) quark flavors** is perhaps less evident. As a matter of fact, the order of the transition in this case is still an open problem and corresponds to the **last question mark** in the zero-density phase diagram of QCD. We argue that establishing the nature of the transition in this case is also a **crucial test** for numerical simulations of lattice QCD, allowing precise estimates of the possible **systematic errors**, related e.g. to the choice of discretized formulation for fermions.

QCD Phase Transition



■ **deconfinement** and **restoration of chiral symmetry** at high temperatures or densities; what are the **properties** of the high-temperature phase (QGP)? what is the **nature** of the transition?

- a **first-order** transition for $\mu = 0$, $T \neq 0$ would mean a real transition in the early universe, perhaps corresponding to cosmological relics.
- a **second-order** transition in the $\mu = 0$ **two-flavor** case (at vanishing quark mass) would probably mean no transition (i.e. a **crossover**) in the general case, with the possibility of a **critical endpoint** in the phase diagram.

Study of the QCD Phase Transition

- **non-perturbative** study through numerical simulations of **lattice QCD** $\Rightarrow$ finite temperature **and** finite density
- for **two flavors**: prediction of critical behavior in the **universality class** of the $O(4)$ **spin model** has not been verified (see F. Wilczek, NPB Proc. Suppl. 2003)
- predicted scaling is observed for **Wilson fermions** but not for **staggered fermions**
- indications of **first-order transition** by M. D'Elia et al., PRD 2005 and arXiv:0707.1987
- if a second-order transition takes place, then **systematic effects** in the simulation may be large around the transition
- in this case, the prediction of **universality** presents a **stringent test** for full-QCD simulations

Lattice QCD at Finite Temperature

$$\mathcal{S} = -\beta \sum_{\square} \frac{1}{N_c} \text{Re Tr } U_{\square} + \sum_{x,y} \bar{\psi}_x K_{x,y} \psi_y$$

In the path integral: **inverse temperature** $\Leftrightarrow$ **temporal extension**

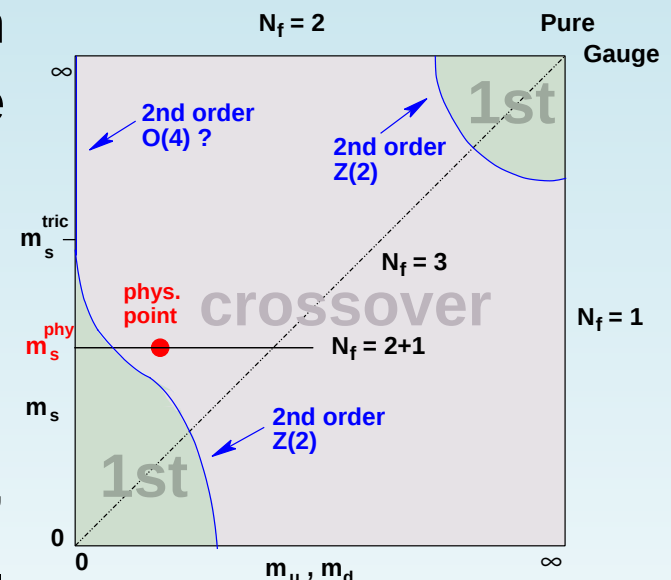
pure gauge (quenched): order parameter given by the Polyakov loop $\langle L \rangle$, breaking of the symmetry $Z(N_c)$

■ $SU(3)$ first order (3-state Potts model)

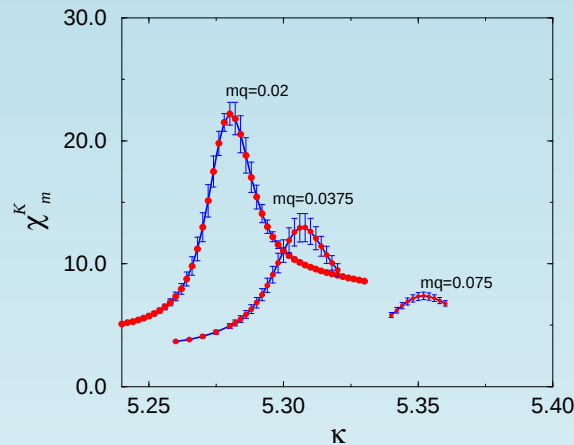
■ $SU(2)$ second order ($3d$ Ising)

dynamical fermions: chiral condensate $\langle \bar{\psi} \psi \rangle$, sensitive to the breaking of the chiral symmetry.

Effective σ model (Ginzburg-Landau theory respecting the chiral symmetry). For two flavors: critical behavior in the universality class of the $O(4)$ spin model (if the transition is second order).



QCD with 2 staggered quark flavors



- order parameter: **Chiral condensate** $\langle \bar{\psi} \psi \rangle$
- magnetic field: quark mass m_q
- reduced temperature: $\tau \sim 6/g^2 - 6/g_c^2(0)$
[from $T = (L_t a)^{-1}$]
- peak of chiral susceptibility: $t_p \sim m_q^{1/\beta\delta}$

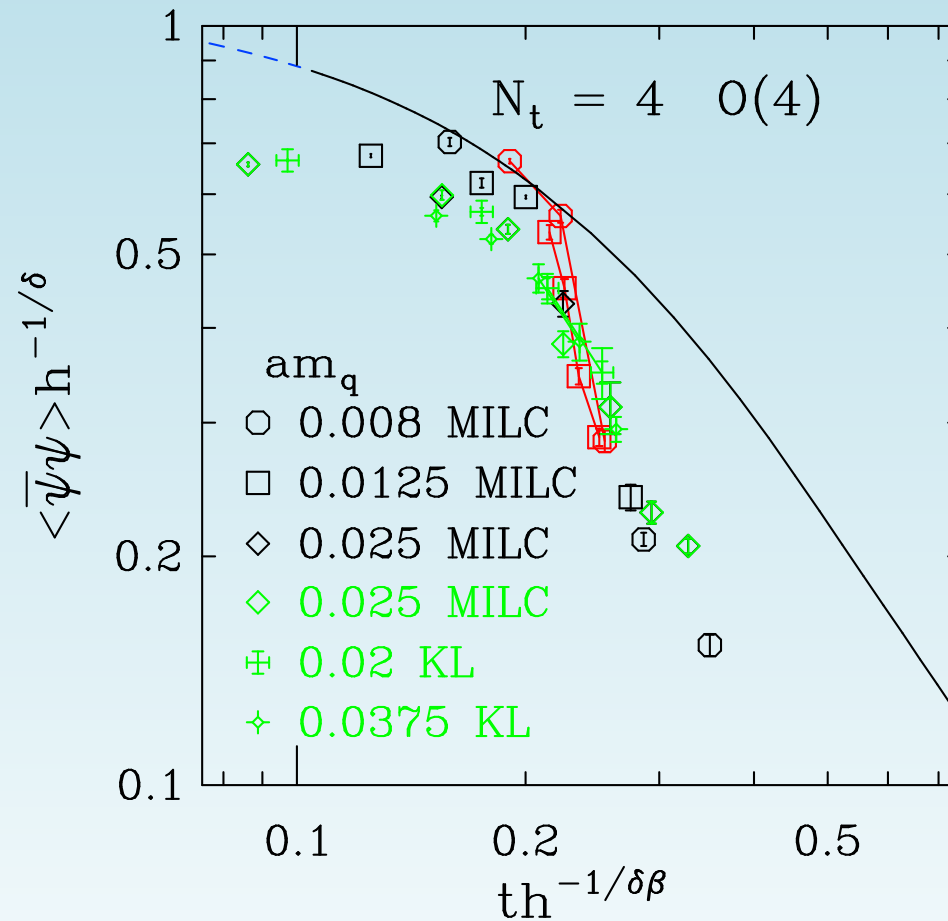
Bielefeld (1994), JLQCD (1998), MILC (2000) → peaks scale with the predicted exponents

3d $O(4)$ model: precise numerical determination of the critical properties. M is described by the **universal** function f_M for all systems in the universality class

$$M/h^{1/\delta} = f_M(t/h^{1/\beta\delta})$$

Engels, **T. M.**, **NPB** '00; Engels et al., **PLB** '01; Cucchieri, **T.M.**, **JPA** 2005.

Scaling Function for 2 Staggered Quarks



Bernard et al., [Phys. Rev. D '00](#)

No Scaling! Possible Problems...

- not **chiral** enough (large quark masses)
- the transition is **not second-order**
- finite-size effects
- systematic effects (e.g. due to use of the **R algorithm**)
- effects due to the **staggered formulation** for quarks

in any case, may perform a **better comparison** by an unambiguous **normalization** of the data [**assuming $O(4)$ behavior** at the transition and using the observed scaling of susceptibility peaks]

In Favor of a First Order Transition

D'Elia, Di Giacomo, Pica (PRD 2005)

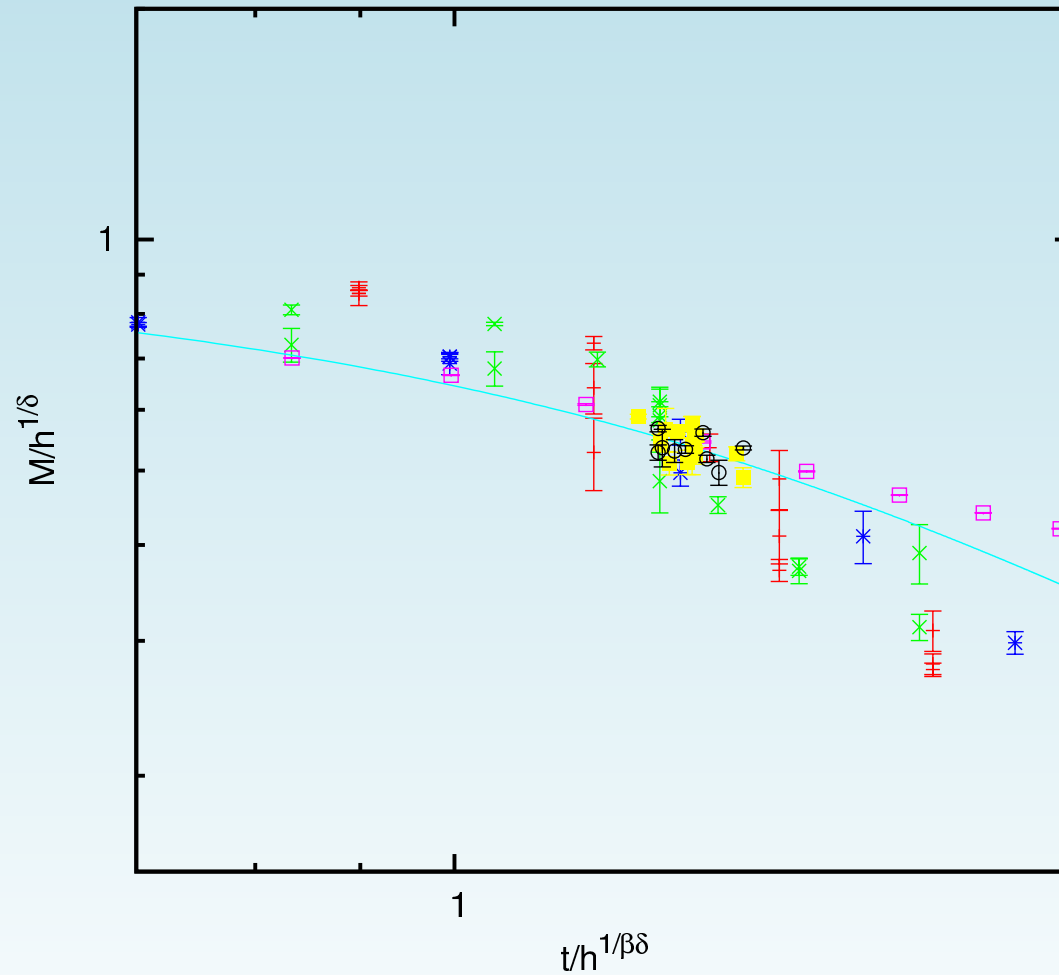
- better definition for temperature in terms of lattice spacing:

$$\tau \sim 6/g^2 - 6/g_c^2(0) + k_m m_q$$

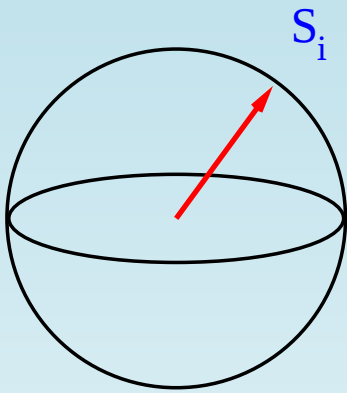
for full QCD $k_m \neq 0$

- data are produced taking finite-size scaling into account
- largest lattices to date (recently: improved fermions, RHMC)
- no evidence for discontinuities
- seems to exclude $O(4)$ scaling and roughly consistent with 1st order scaling

∞ -Volume Scaling for New Definition of τ

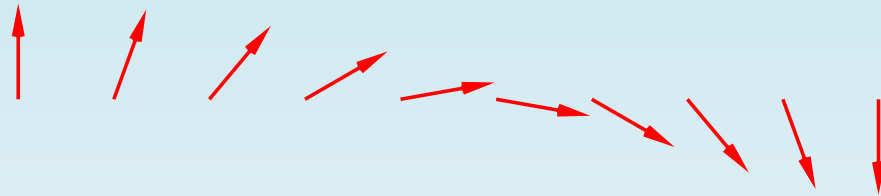


O(N) Spin Models



$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - \mathbf{H} \cdot \sum_i \mathbf{S}_i$$

continuous symmetry $\rightarrow$ spin waves



- no spontaneous magnetization in $d = 2$
- Goldstone-mode effects in $d = 3$: diverging susceptibility and strong finite-size effects for all $T < T_c$ when $H \rightarrow 0$

$$\chi_T \sim 1/H; \quad \chi_L \sim H^{-1/2} + \dots$$

Universality

- **Critical Exponents:** for Magnetization M (order parameter) and Susceptibility χ

$$M_{h=0} \sim |t|^\beta, \quad \chi_{h=0} \sim |t|^{-\gamma}, \quad M_{t=0} \sim h^{1/\delta}$$

where $t = (T - T_c)/T_0$, $h = H/H_0$.

[Exponents are known for the 3d $O(4)$ -model]

- **Scaling Function:** given T_0 and H_0 , M is described by the **universal function** f_M for all systems in the universality class

$$M/h^{1/\delta} = f_M(t/h^{1/\beta\delta})$$

Normalized f_M calculated for 3d $O(4)$ -model

Engels, **T. M.**, Nucl. Phys. B572, 289 (2000)

(Infinite-Volume) Scaling

Scaling hypothesis is given by

$$F_s(t, h) = b^{-d} F_s(b^{y_t} t, b^{y_h} h),$$

where b is a scaling factor, t and h are the reduced temperature and field and y_t , y_h are related to the usual critical exponents.

Scaling Function for the Magnetization

$$M/h^{1/\delta} = f_M(t/h^{1/\beta\delta})$$

or, equivalently,

$$y = f(x) \quad \text{with} \quad y = h/M^\delta, \quad x = t^\beta/M.$$

Ansatz (Wallace and Zia '75):

$$x = -1 + a y^{1/2} + b y + c y^{3/2} + \dots$$

Pseudo-Critical Line ($H \neq 0$)

Defined by the (finite) peaks of χ , corresponding to the divergence for $H = 0$, $T = T_c$.

Scaling Function for χ

$$\chi = \partial M / \partial H = (1/H_0) h^{1/\delta-1} g(t/h^{1/\beta})$$

with $g(z)$ **universal** (related to f_M).

Peak for each fixed h : $t_p = z_p h^{1/\beta\delta}$

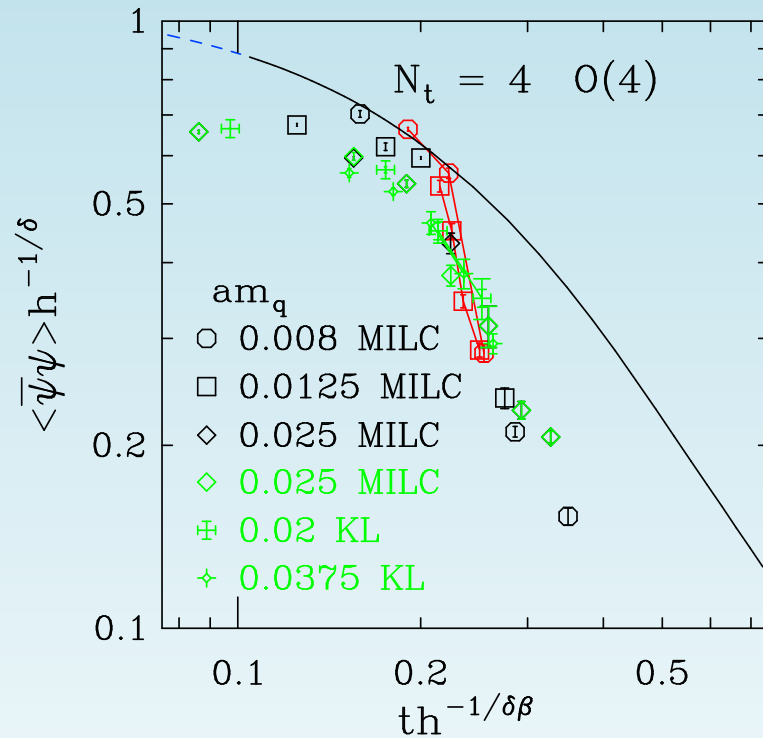
$$M_p = h^{1/\delta} f_M(z_p), \quad H_0 \chi_p = h^{1/\delta-1} g(z_p).$$

Thus the **pseudo-critical line** is given by the **universal constants** z_p , $f_M(z_p)$, $g(z_p)$. For the 3d $O(4)$ -model $z_p = 1.33(5)$

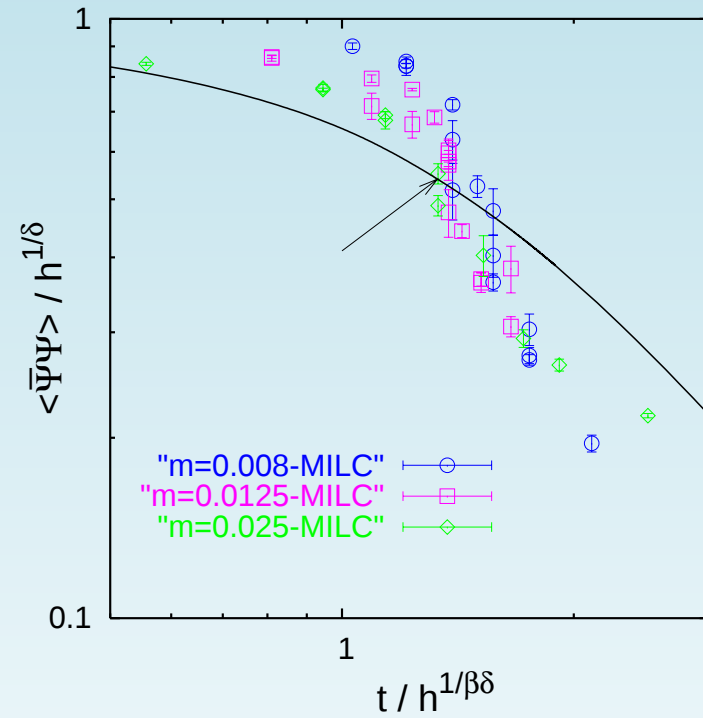
Engels et al., Phys. Lett. B514, 299 (2001)

Scaling Function

After Normalization



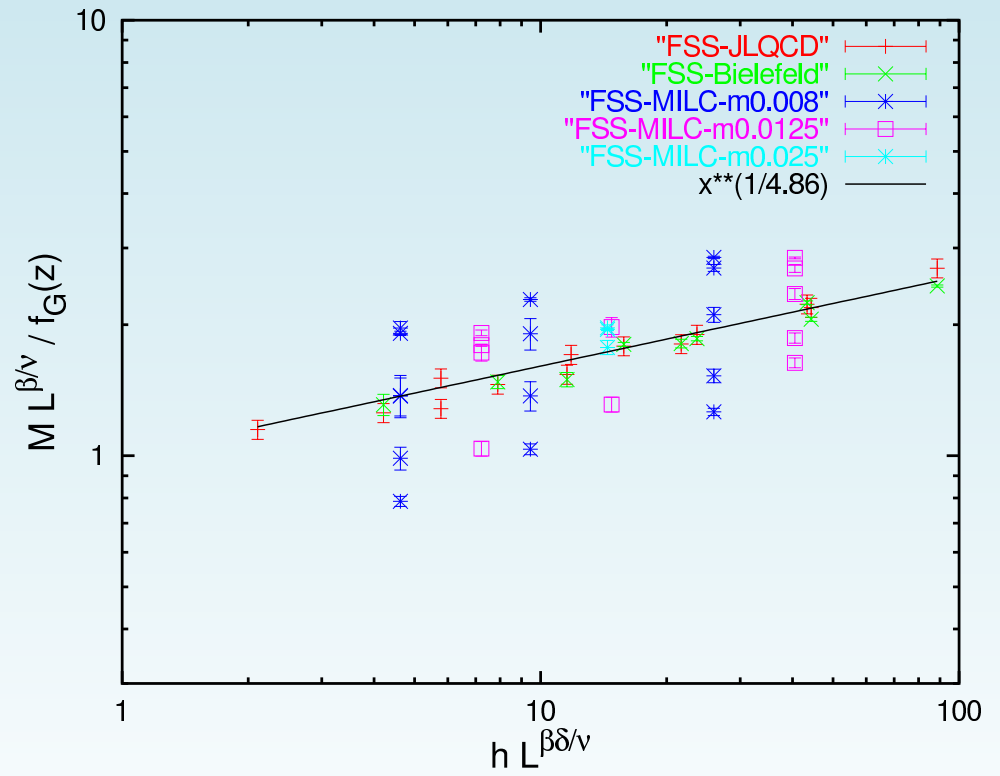
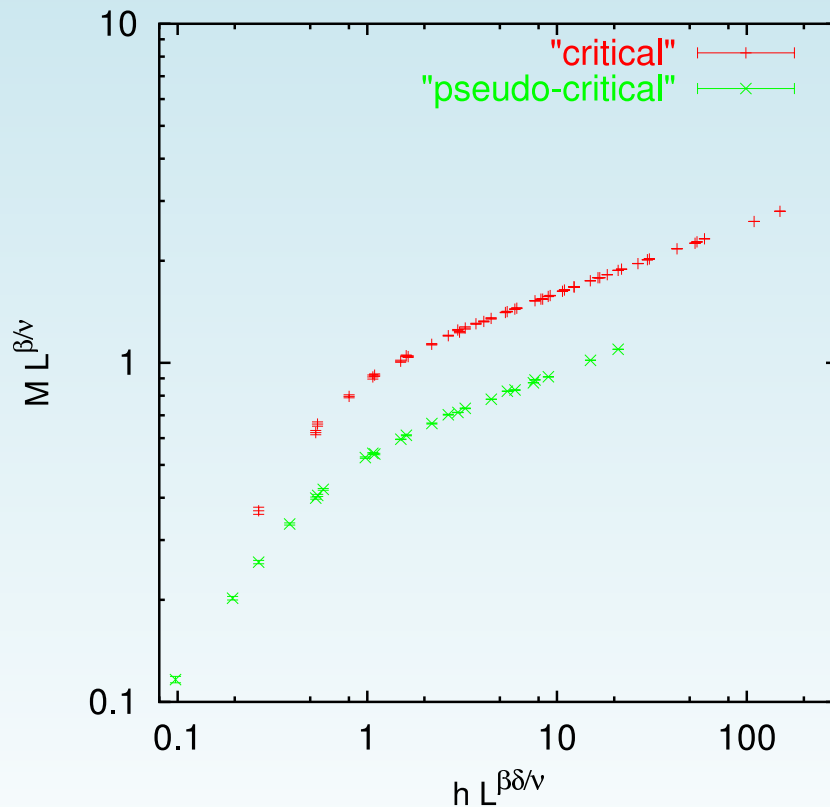
Bernard et al., Phys. Rev. D '00



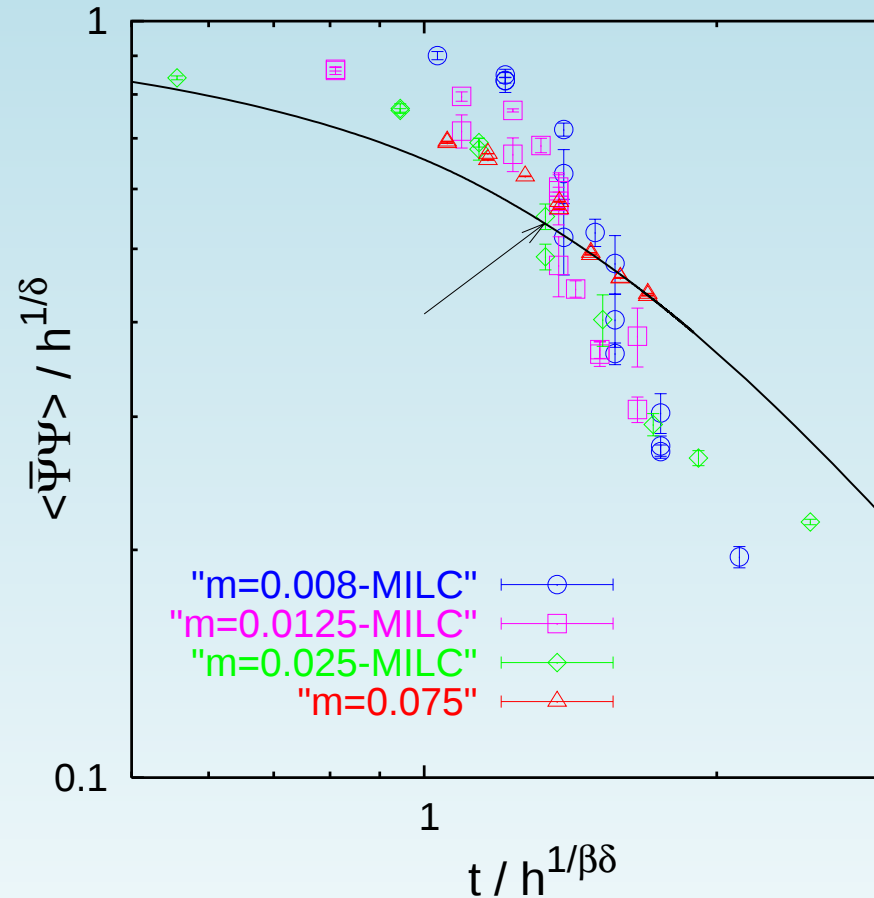
T.M., Nucl. Phys. A '02

Finite-Size Scaling

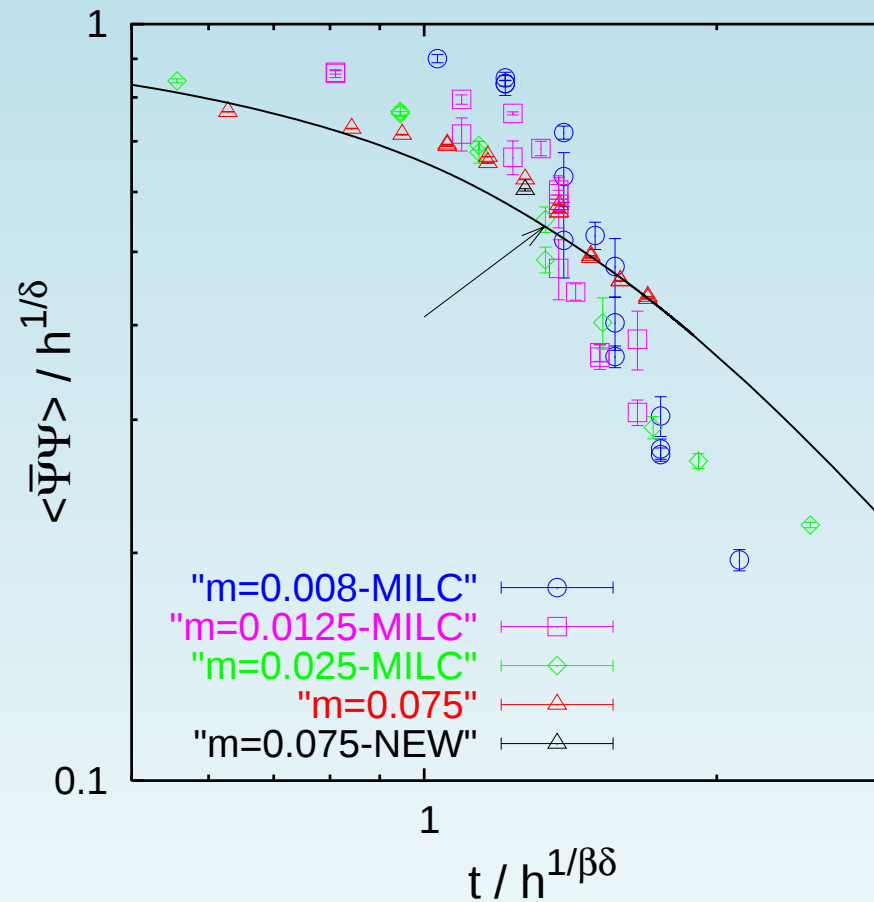
$F_s(t, h, L) = L^{-d} Q(L^{1/\nu} t, L^{\beta\delta/\nu} h) \Rightarrow M = L^{-\beta/\nu} Q_M(h L^{\beta\delta/\nu}),$
 for fixed $z = t/h^{1/\beta\delta}$ (e.g. $z = 0$, $z = z_p$). **Note:** for $L \rightarrow \infty$ we have
 $M = h^{1/\delta} f_M(z)$, thus $Q_M(u) \rightarrow f_M(z) u^{1/\delta}$ for large u



Including New Data at Larger Masses



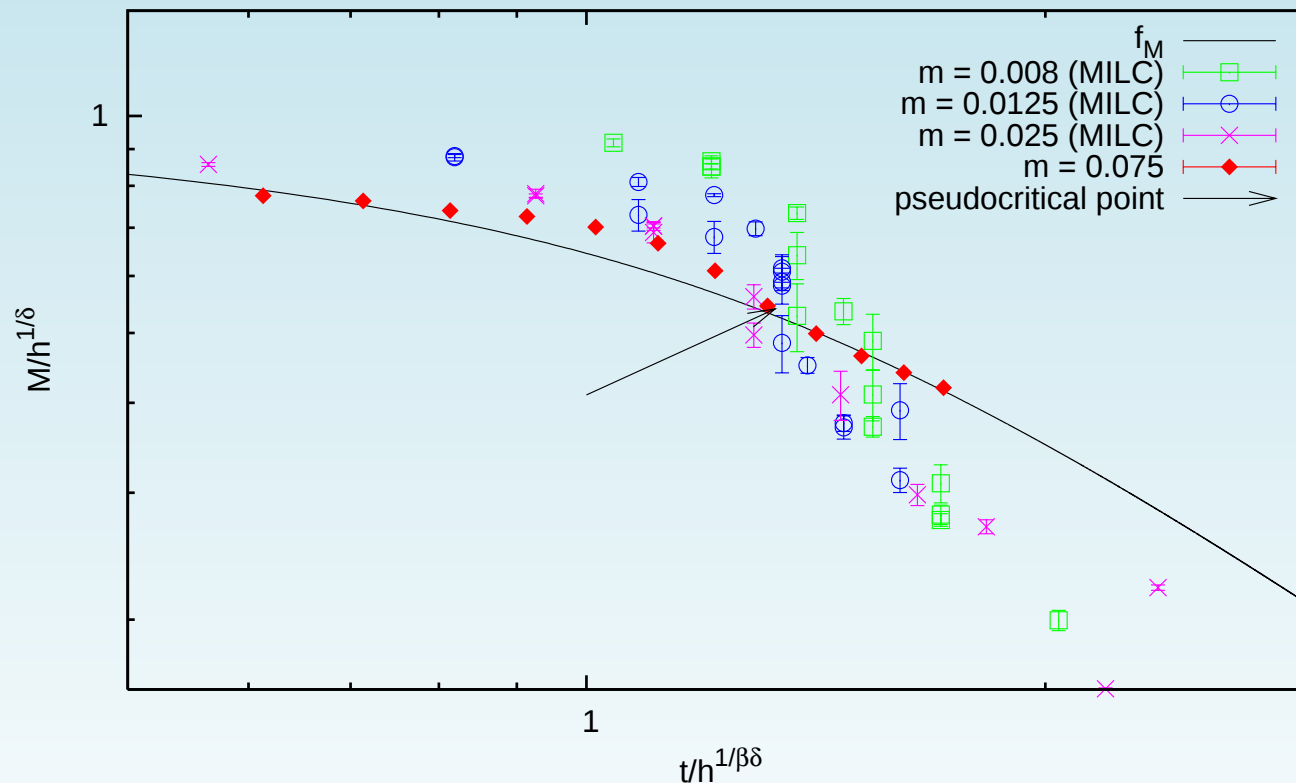
T.M., Cucchieri, AIP Conf. Proc. (2004)



T.M., AIP Conf. Proc. (2005)

In Favor of a Second-Order Transition

Plot of scaling function including new data at the largest mass
(and smaller δt), combined with older MILC data



T.M., PoS (LATTICE 2007)

Conclusions

- By an unambiguous normalization of QCD data for 2 staggered fermions the predicted $O(4)$ scaling seems to be achieved for larger quark masses $\Rightarrow$ Agreement with Wilson case (second-order transition)
- The universal scaling range around the pseudo-critical point z_p seems to be rather large, with data at smaller masses describing a wiggle around z_p . Simulation points for $m = 0.075$ show good scaling, even to the right of z_p
- Using a significantly smaller integration step in the R-algorithm the deviations for $m = 0.075$ decrease visibly; results suggest that the (large) deviations at small masses might be partly due to systematic errors in the simulations
- 2-flavor case is especially suitable for testing better algorithms and fermion formulations